MECHANICS OF BILLIARDS, AND ANALYSIS OF WILLIE HOPPE'S STROKE

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January 17, 1942

January 6, 1947: Additional Notes.

Since writing the manuscript five years ago, some additional facts have come to light, and should be mentioned here.

At the bottom of the page, page 26, it is concluded that the cue ball initial velocity, in the nine-cushion shot, is no more than in the break shot. This is wrong. Hoppe and Cochran both say that the nine-cushion shot takes a much harder stroke. Although I have never managed to make this shot, my attempts at it force me to agree.

I went wrong in my analysis by assuming that the flash interval in the Life photographs was the same for all shots. As it turns out, the apparatus (which I have learned was built at Bell Laboratories by a former student of mine!) was adjustable: for any one shot, the best flash interval for recording that shot could be used.

On page 39, I stated certain conclusions about the bridge, and tightness of the bridge. Since studying neuromuscular phenomena, and after recording my own stroke and analyzing it, I conclude that the most important function of the professional's tight bridge is to furnish a constant resistance to cue movement. This, in turn, requires him to shoot tetanically. That is, the stroking muscles are in a constant state of contraction while accelerating the cue. My theory is that only in this was can one master the "velocity" part of the game, and reliably impart to the cue ball the desired velocity, in order to play position.

With an open bridge, the amateur's cue meets with very little frictional resistance as it goes forward. I believe that therefore, he shoots ballistically. His muscles yank quickly on the cue at the very start of the stroke. The accelerating phase is over in a very short distance; and thereafter, his hand is riding with the cue at nearly constant velocity until the cue strikes the ball. I maintain that no one can hope, with ballistic contraction of the muscles (a quick yank), that he can control cue velocity at impact as well as can be done by using tetanic contraction.

A. D. Moore

FOREWORD.

According to Willie Hoppe, the game of billiards attracts some 10,000,000 players in the United States alone. Four factors combine to make the present time a notable one in the long history of the game. This is especially true of three-cushion billiards.

First, of the several billiard games, three-cushion billiards has become the choice of the public and the professionals in recent years. Second, at the peak of a career unequalled in any sport, both for length of career and consistency in winning championships, Willie Hoppe adopted three-cushion as his game. Third, photographic records of Hoppe's play have recently become available, through the repetitive flash photographs taken for Life Magazine by Gjon Mili. Fourth, Hoppe recently published his excellent book, "Billiards as it should be played", and thereby made a wealth of information available.

In spite of the fact that scientists and engineers are fond of the game, it does not appear that analysis of the game's phenomena has been made. Such an analysis is long overdue.

The studies presented herein are themselves an outgrowth of several favoring factors. First, the writer has played three-cushion billiards for perhaps 35 years. Second, the writer had the rare privelege of playing an exhibition game with Willie Hoppe at the University Club, Ann Arbor, in the fall of 1941; and later at dinner, he took full advantage of the opportunity to put many questions to Mr. Hoppe. Third, the writer was already acquainted with a number of Life Magazine executives, through working with them on the Michigan-Life Conference on New Technologies in Transportation in 1939; it was therefore easy to persuade Mr. Wesley L. Bailey of the Life Executive Offices to lend the writer Life's file of 36 Mili flash photographs of Hoppe's shots.

These studies began in the usual innocent way. The photographs made it appear that a little casual study, in odd moments, would reveal something about the stroke motions of the cue. The writer had no intention of letting himself in for all that developed, as one discovery pointed to the next. But one thing did lead to another, and there was no mental peace possible until the several major puzzles had been solved. The writer has long been in debt to the game for the fine recreation it has furnished. He is now still further indebted for the pleasure derived from analysing some of the games puzzles and problems, from the mechanics standpoint.

In order to analyze Hoppe's stroke, it was necessary to solve the cue-to-ball impact problem; but in order to do that, it was first necessary to measure the coefficient of restitution. This is why, in the paper, the coefficient is covered at the outset. The paper is in five sections:

I. Experimental Determination of the Coefficient of Restitution.
II. Cue-to-Ball Impact: Solution of the Problem.
III. Analysis of Willie Hoppe's Three-Cushion Stroke,
IV. The Course of the Ball: Analysis of some of the Phenomena.
V. Further Notes.

SECTION I. EXPERIMENTAL DETERMINATION OF THE COEFFICIENT OF RESTITUTION.

Coefficient of Restitution.

When there is impact between two bodies, and in such manner that the line of action passes through the centers of the two masses, it is called Direct Central Impact. If both bodies were perfectly elastic, there would be no loss of energy; and it has long since been shown that the relative velocity after impact equals the relative velocity before impact,- irrespective of the amounts of the two masses.

With imperfectly elastic bodies, there is a loss of energy, and relative velocity after impact is less than that before impact. If the objects are numbered 1 and 2; if their velocities are u_1 and u_2 before impact and are v_1 and v_2 after impact, the so-called Coefficient of Restitution is defined as

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The coefficient must be determined experimentally. Very few coefficients have been published. A search of the mechanics literature revealed only four sources*, and they collectively gave only these coefficients: glass, 0.94 to 0.95; ivory, 0.81 to 0.89; cast iron, 0.50 to 0.65; steel, 0.55 (certainly, not hardened steel!); cork, 0.55; wood, 0.5; lead, 0.15 to 0.20; putty and clay, zero.

The sources are: "Handbook of Engineering Fundamentals," Eshbach; "Textbook of Mechanics", Martin; "Applied Mechanics", Poorman; and "Analytical and Applied Mechanics", Clements & Wilson.

A cue is a wooden shaft, with a piece of bone at the front end, and a leather tip ahead of that. If wood is at e = 0.5, and if leather is assumed to be no better that cork (0.55), then, even though ivory is around 0.85, one inclines to the guess that the cue-to-ball combination may be no better than 0.5. This turns out to be far short of the true value. As a matter of fact, a good professional tip, as Hoppe states, is quite hard. Also, the impact is in line with the grain of the wood: furthermore, it is hardwood.

Experimental Set-up.

At any rate, the writer experimentally determined e for the case in hand. The set-up was made and the readings were taken in one afternoon (11-25-1941). The simplicity of the set-up, the ease of making observations, and the accuracy secured indicate that someone could, without much effort, add greatly to our list of coefficients.

The cue and ivory ball were hung by light steel wires beneath a horizontal 10-inch board. String was wrapped around the ball, and held in place with Scotch tape. Loops in the string permitted two wires to be attached. The wires went to screw-eyes put in the edges of the board, across from each other. Similar pairs of wires (two pairs) suspended the cue for direct central impact, The vertical radii were all 20 inches. When both bodies were at rest, they grazingly touched.

The writer's brass-jointed cue was used. Its length is 57.5 inches (which is over an inch too long, by the way!), with the "balance" at 16.5 inches from the butt end. It had just returned from the factory, and had a new hard professional-type tip, in perfect condition.

Cue weight, 1.390 lb. Ball weight, 0.455 lb. Temperature, 72 F.

Method.

The cue was pulled back to a backstop, then released. Four values of backswing were used. The horizontal values of backswing, and of forward swing of cue and ball after impact, were measured. These measurements were made by looking vertically downward at scales laid horizontally. Readings for any one condition were taken only after practice, and after consistently repeating swings were observed. Table X gives the data.

Table I. Impact and Coefficient of Restitution

R = vertical radius = 20 in.
P_a = initial backswing of cue, inches, measured horizontally
P_b = forward swing " " " " " after impact.
P_c = " " " ball, " " " " "

u_1 = cue vel. before impact
v_1 = " " after "
v_2 = ball " " "

Readings, Inches horizontally			Height, inches. $H = R + \sqrt{R^2 - P^2}$			Velocities, ft. per sec. $v = \sqrt{2gh}$ $\left\{ \begin{array}{l} g = 32.2 \\ h \text{ in ft.} \end{array} \right.$			$\frac{v_2 - v_1}{u_1} = e$
P_a	P_b	P_c	H_a	H_b	H_c	u_1	v_1	v_2	
2.0	1.1	2.72	0.100	.0302	.186	.734	.402	.999	0.814
4.0	2.18	5.45	0.404	.119	.724	1.47	.800	1.97	0.795
6.0	3.2	8.1	0.921	.258	1.71	2.22	1.176	3.03	0.845
8.0	4.4	10.7	1.67	.491	3.09	2.99	1.62	4.06	0.783

In Fig. 1, v_1 and v_2 are plotted on log paper against u_1 . Ideally, the two lines would be straight, parallel, and sloped at 45 degrees, The lines were so drawn, and they show excellent agreement among the data. Using the lines as drawn, ani these values from the lines: $u_1 = 1.00$, $v_1 = 0.54$, $v_2 = 1.35$, a suitable value of e is found by ..

$$e = \frac{v_2 - v_1}{u_1} = \frac{1.35 - 0.54}{1.00} = 0.81$$

The data are so consistent that high accuracy must have been obtained, Swing losses were looked into. With cue swinging freely, an amplitude of 8 inches decayed to 7 inches in 7 swings. It is concluded that e was determined to within 4%. Furthermore, the velocity range used covers a large majority of three-rail shot velocities.

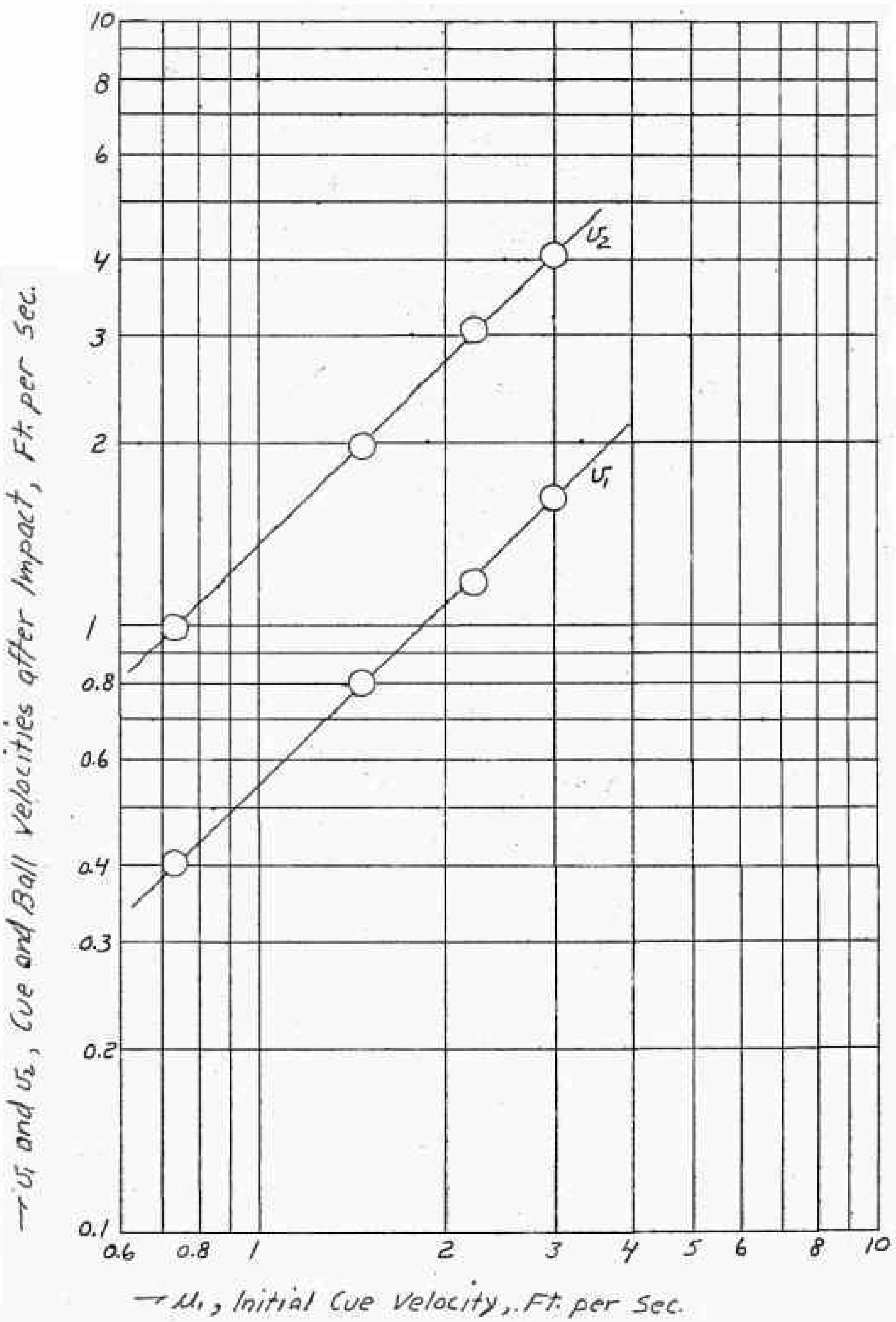

v_2
v_1
10
8
6
4
3
2
1
0.8
0.6
0.4
0.3
0.2
0.1
0.6
0.8
1
2
3
4
5
6
8
10
→ v_1 and v_2, Cue and Ball Velocities after Impact, Ft. per Sec.
→ u_1, Initial Cue Velocity, Ft. per Sec.

Fig. 1

SECTION II. CUE-TO-BALL IMPACT: SOLUTION OF THE PROBLEM.

After consulting a number of experts in the field of mechanics and searching a good deal of the mechanics literature, the writer found that no one appears to have published any solution for the off-center impact problem arising when a ball is cued. It was then necessary to work out the solution. As far as we know, this is the first solution made for the case of a rod, going endwise, striking a sphere with Direct Eccentric Impact.

In the following pages, the general solution is given, followed by numerical solutions and derived pertinent data for the whole range of cases normally occurring in billiards. For the numerical solutions, strictly typical basic data, as they occur in typical billiard cues and balls, were used.

A summary of important findings will be given here, rather than at the end of the Section.

<u>Summary</u>.

A typical billiard ball is 2.4 inches in diameter. It is typically cued at a point anywhere between dead center (h = zero) to a usual extreme of 0.6 inches from center (h = 0.6). The distance from dead center is <u>not</u> measured around the curve of the ball. For this off-center distance, h, see Fig. 2.

As h is varied through the above range, starting from zero, the following statements are true:

1. Ball energy in % of cue energy drops from 66.7 to 51.7%.

2. It is a low-loss impact: loss, in % of cue energy, ranges from 10.0 down to 7.4%. (Table III)

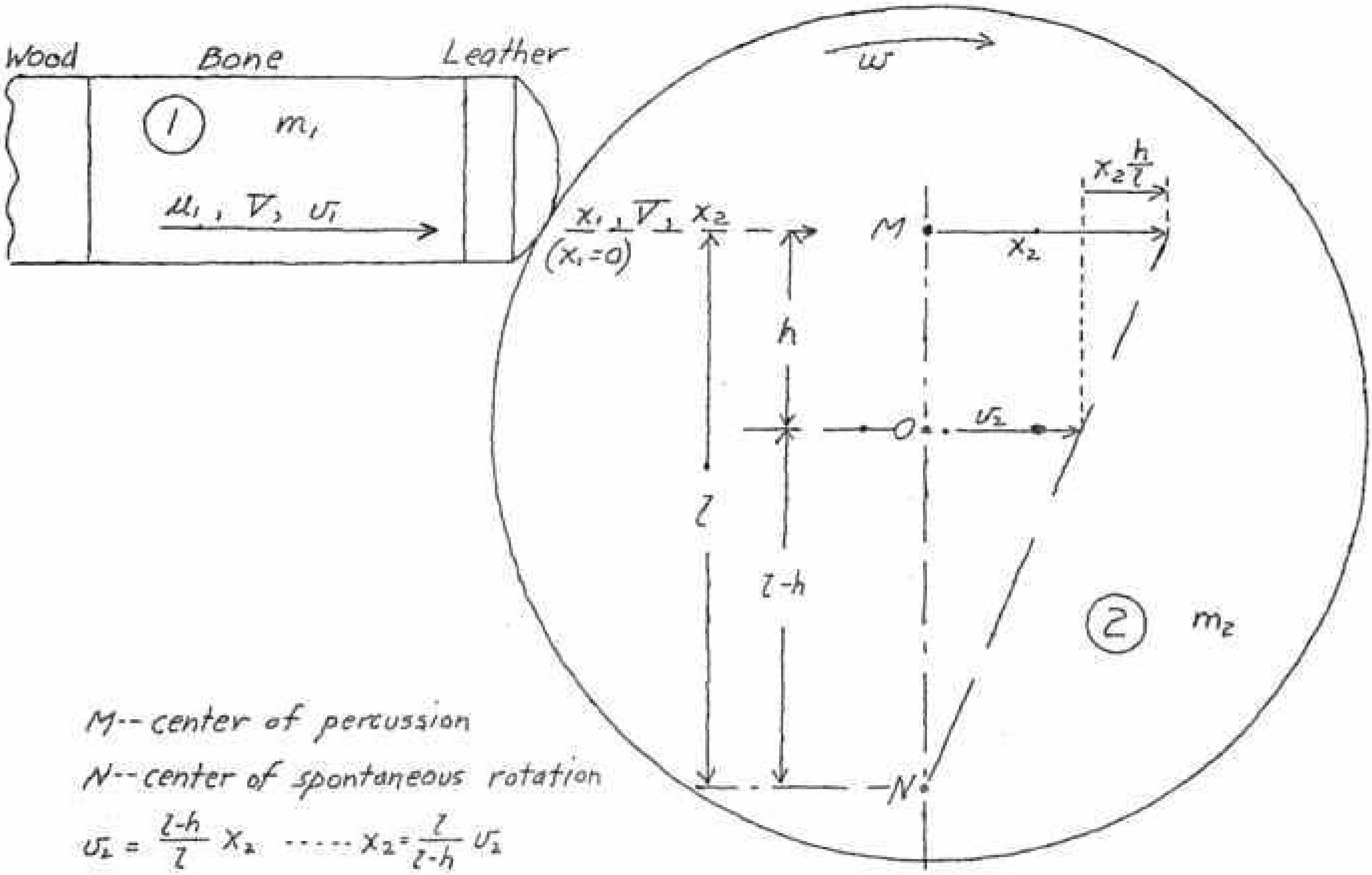
3. Rotational energy in % of total ball energy ranges from zero, up to 38.4% (Table II)

4. The ration of post-impact cue velocity to ball velocity ranges from 0.373 to 0.720 (Table II)

5. The ratio of post-impact to pre-impact cue velocity ranges from 0.483 to 0.643 (Table II)

General: the high possible content of rotational energy is all-important in determining the behaviour of the ball in flight,- as is brought out in Section IV.

The value of e as found experimentally, 0.81, is used in the solutions. It was determined within a range of velocities that covers most of the velocities used in three-cushion shots. The following solutions and data should therefore apply quite closely to billiard situations in which good cues, tips and balls are used.



M-- center of percussion

N-- center of spontaneous rotation

$v_2 = \frac{l-h}{l} x_2$ ----- $x_2 = \frac{l}{l-h} v_2$

$\omega h = x_2 \frac{h}{l}$

$\omega, V,$ simultaneous -- occur during impact when common velocity is achieved at point of contact.

$\omega = \frac{V}{l}$

Fig. 2

Direct Eccentric Impact

Direct eccentric impact: cue moving in line with its own center line, but hitting the ball off center.

Notation: much as in Martin, "Textbook of Mechanics."

P = varying forces of Compression
R = " " " restitution
V = common velocity through center of percussion at end of compression
W = angular velocity of ball, simultaneous with V
e = coefficient of restitution
κ = radius of gyration
other symbols as in Fig. 2

COMPRESSION

Cue --- $-\int P dt = m_1 v\Big]_{u_1}^{V} = m_1 (V - u_1)$ ---------------- (1)

Ball Translat. -- $\int P dt = m_2 v\Big]_{0}^{\frac{l-h}{l}V} = m_2 \frac{l-h}{l} V$ ---------------- (2)

Ball Rotat. --- $Ph = \kappa^2 m_2 \, d\omega/dt$

$$h\int P dt = \kappa^2 m_2 \omega\Big]_0^{W} = \kappa^2 m_2 W, \quad \int P dt = \frac{\kappa^2 m_2 W}{h}$$ ---------- (3)

RESTITUTION

Cue --- $-\int R dt = m_1 v\Big]_{V}^{v_1} = m_1 (v_1 - V)$ ---------------- (4)

Ball Translat. --- $\int R dt = m_2 v\Big]_{\frac{l-h}{l}V}^{\frac{l-h}{l}x_2} = m_2 \frac{l-h}{l}(x_2 - V)$ ---------------- (5)

Ball Rotat. --- $h\int R dt = \kappa^2 m_2 \omega\Big]_{W}^{\omega} = \kappa^2 m_2 (\omega - W)$, --- $\int R dt = \frac{\kappa^2 m_2}{h}(\omega - W)$ ----- (6)

Next, re-list the 6 equations, substitute for ω, W, change signs in (1) and (4)

$\int P dt = m_1 (u_1 - V)$ ---- (1) $\qquad \int R dt = m_1 (V - v_1)$ ------ (4)

$\int P dt = m_2 \frac{l-h}{l} V$ ---- (2) $\qquad \int R dt = m_2 \frac{l-h}{l}(x_2 - V)$ ---- (5)

$\int P dt = m_2 \frac{\kappa^2}{hl} V$ ---- (3) $\qquad \int R dt = m_2 \frac{\kappa^2}{hl}(x_2 - V)$ ---- (6)

$m_2 \frac{l-h}{l} V = m_2 \frac{\kappa^2}{lh} V$ from (2) and (3)

$(l-h)h = \kappa^2$, to be used later ---------------- (7)

From (1) and (2), $m_1 u_1 - m_1 V = m_2 \frac{l-h}{l} V$, $\therefore V = \dfrac{m_1 u_1}{m_2 \frac{l-h}{l} + m_1}$ ------ (8)

From (4) and (5), $m_1 V - m_1 v_1 = m_2 \frac{l-h}{l} x_2 - m_2 \frac{l-h}{l} V$, $V = \dfrac{m_2 \frac{l-h}{l} x_2 + m_1 v_1}{m_1 + m_2 \frac{l-h}{l}}$ ------ (9)

From (8) and (9), $m_1 u_1 = m_2 \frac{l-h}{l} x_2 + m_1 v_1$ ---------------- (10)

Now e takes the form --- $e = \frac{x_2 - v_1}{u_1}$, $e u_1 = x_2 - v_1$, $x_2 = v_1 + e u_1$ ------ (11)

Substitute for x_2 in (10).

$m_1 u_1 = m_2 \frac{l-h}{l}(v_1 + e u_1) + m_1 v_1$ ---- $m_1 u_1 = m_2 \frac{l-h}{l} v_1 + e m_2 \frac{l-h}{l} u_1 + m_1 v_1$

$\left(m_1 - e m_2 \frac{l-h}{l}\right) u_1 = \left(m_1 + m_2 \frac{l-h}{l}\right) v_1$

$$u_1 = \frac{m_1 + m_2 \frac{l-h}{l}}{m_1 - e m_2 \frac{l-h}{l}} v_1 \qquad \text{—————— (12)}$$

CASE. Solve for the case: ball cued at such a value of h that, if struck directly above center, the spontaneous center of rotation is at point of ball contact with table. Then,-

$l - h$ = radius = 1.2 in. Using (7), and K^2 for sphere $= \frac{4r}{10} = .4(1.2) = 0.576$,

$l - h = K^2$

$h = (0.576) \div 1.2 = 0.480$

$l = 1.68$

$(l-h) \div l = 0.715$

$e = 0.81$, from Section I.

$m_2 = 1$, $m_1 = 2.5$ ... relative values

substituting values in (12),

$u_1 = \frac{2.5 + 0.715}{2.5 + 0.81(.715)} v_1 = \frac{3.215}{1.921} v_1$

$\underline{u_1 = 1.68\ v_1}$ --- $\underline{\underline{v_1 = 0.595\, u_1}}$

$x_2 = v_1 + e u_1$ -- (11)

$= v_1 + 0.81(1.68\, v_1) = (1 + 1.36)\, v_1$

$\underline{\underline{x_2 = 2.36\ v_1}}$

v_2, ball center velocity, is

$v_2 = \frac{l-h}{l} x_2 = 0.715(2.36\, v_1)$

$\underline{\underline{v_2 = 1.69\ v_1}}$ ---- $\underline{\underline{v_1 = 0.593\ v_2}}$

<u>OTHER CASES</u>, for the range $h = 0$ to $h = 0.6$ were also worked. The results are given in Table II.

Study of Translational and Rotational Energies, letting h vary.

If x_2 is the total velocity of M (Fig. 2)

$v_2 = x_2 \frac{7-h}{7}$, v_2 being translational velocity of ball center

$x_2 \frac{h}{7}$ is the rotational component of M about ball center.

$\omega h = x_2 \frac{h}{7}$

$\omega = \frac{x_2}{7}$

Total kinetic energy of ball, E, is

$E = \frac{I\omega^2}{2} + \frac{m_2 v_2^2}{2}$ where I, moment of inertia, sphere, is $I = \frac{2m_2 r^2}{5} = \frac{2m_2(1.2)^2}{5} = 0.576\, m_2$

$E = \frac{0.576\, m_2 \omega^2}{2} + \frac{m_2}{2}\left[\frac{7-h}{7} x_2\right]^2$ ... drop $\frac{m_2}{2}$

$E \propto 0.576\left(\frac{x_2}{7}\right)^2 + \left[\frac{7-h}{7} x_2\right]^2$ ... drop x_2^2

$E \propto \frac{0.576}{7^2} + \left[\frac{7-h}{7}\right]^2$

But $(7-h)h = K^2$ — — — — (7)

And $K^2 = 0.576$

$\therefore\ 7-h = \frac{0.576}{h}$

Energies, energy ratios, losses, and velocity ratios can now be computed. The results are entered in Tables II and III.

Table II. Energies, Velocity Ratios. Relative values of m_1, m_2 used.

$h=0$, ball cued at dead center

$h=0.48$, the case worked in detail above.

h	$z-h$	z	$(z-h)/z$	Values are $\propto$ to real energies: Rotation R, $\frac{0.576}{z^2}$	Translat. T, $\left(\frac{z-h}{z}\right)^2$	Both $R+T$	Ratio $\frac{R}{R+T}$	Velocity Ratios: $\frac{v_1}{v_2}$	$\frac{v_1}{u_1}$
0	∞	∞	1.000	0	1.000	1.000	0	.3735	.4828
0.1	5.76	5.86	.983	.0168	.967	.984	01705	.383	.459
0.2	2.87	3.07	.935	.0611	.874	.935	0655	.412	.508
0.3	1.92	2.22	.865	.1168	.748	.865	.135	.460	.535
0.4	1.44	1.84	.782	.170	.611	.781	.218	.525	.568
0.48	1.20	1.68	.715	.204	.511	.715	.286	.593	.595
0.5	1.15	1.65	.696	.211	.485	.696	.303	.615	.605
0.6	0.96	1.56	.615	.236	.379	.615	.384	.720	.643

Table III. Further Energy Studies. All values $\propto$ to actual energies

$\left(\frac{v_2}{v_1}\right)\left(\frac{v_1}{u_1}\right) = \frac{v_2}{u_1}$ ... at $u_1 = 1$, cue initial energy $\propto m_1 u_1^2 \propto m_1 = 2.5$

h	$\frac{v_2}{u_1}$	Ball translational energy $\propto v_2^2$	Ball total energy $\propto$ (Ball translational)$\left(\frac{R+T}{T}\right)$	Cue final energy $\propto m_1 v_1^2 = 2.5\, v_1^2$	Total final system kinetic energy	Loss of energy (from a smoothing curve)	Loss, in % of cue initial energy	Ball energy in % of cue init. energy
0	1.292	1.672	1.672	.583	2.255	.250	10.0	66.7
0.1	1.278	1.631	1.660	.598	2.258	.240	9.6	66.5
0.2	1.234	1.524	1.630	.646	2.276	.229	9.2	65.3
0.3	1.162	1.350	1.560	.715	2.275	.218	8.7	62.5
0.4	1.081	1.170	1.494	.808	2.302	.206	8.3	59.8
0.48	1.003	1.006	1.408	.885	2.293	.198	7.9	56.3
0.5	.983	.966	1.388	.915	2.303	.196	7.8	55.5
0.6	.893	.797	1.291	1.033	2.324	.184	7.4	51.7

SECTION III. ANALYSIS OF WILLIE HOPPE'S STROKE.

In billiards, the stroke is of supreme importance. For a small fraction of a second, the cue is in contact with the ball. After that brief contact at impact, the player is out of the picture: physical laws take over. If a player knows the shots and handles the cue properly, he wins. If not, he loses. Hoppe's handling of the cue is superb. An analysis of what this master player does with the cue is of great interest. An analysis, as complete as is now possible, is presented in the following pages.

What the Photographs Show.

In the Mili-Life Magazine flash photographs of Hoppe's play, the total flight of the ball is flashed, or pictured, from perhaps 25 times at the least to perhaps 70 times at the most. Thus, a wealth of information about the course of the ball is available. Some of it is used in Section IV.

But as to the cue - the stroke takes up only a brief part of the total time used to record a given shot. Relatively few positions of the cue along its stroke are therefore available. In fact, only 13 of the photographs yield reliable information on cue position; and in all these, only the positions after impact are available. No pre-impact parts of the stroke (except the start) are shown with certainty.

As to yielding up their information, the photographs are rather reluctant. A job of work had to be done to dig out the facts. All of the photographs are, of course, portrayals in perspective. The rules of perspective had to be applied, so that all dimensions could be corrected for foreshortening effect. The presence of the diamonds along the rails and the known dimensions of the table permitted the actual distances to be computed. Most of the measurements cited in Tables IV and V are correct to within 3%. In fact, in several cases, the known diameter of the ball was checked to within 2%.

The sketch in Fig. 3 shows, in general, what may be seen and measured in the 13 photographs. In 8 of the photographs, only one flash of the cue showed, between impact and end of stroke. But fortunately, in one photograph, three flashes showed; and in each of two other cases, two flashes were caught.

The Shots Analyzed.

Data on five selected shots are grouped in Table IV. These five shots were so grouped because S_1', the stroke after impact, was within the fairly narrow and very usual range of 0.70 to 0.94 feet. The other important feature common to all five shots was that h, the distance off-center of cueing the ball, was in the neighborhood of 0.48 in all five. Table IV also includes notations on ease or hardness of stroke; and it refers to the diagrams of the shots, as found in "Billiards..." by Hoppe. Data on the remaining 8 shots are likewise given in Table V.

In both tables, the distances from point of impact of cue or ball

Table IV. Five Selected Shots.

All cued much alike as to distance off center --- h about 0.48 inches.
Strokes after impact much alike --- 0.7 to 0.94 feet.

Measurements corrected for camera foreshortening.
" correct to within 3% (several cases, balls checked to 2%)

Ball velocities corrected for loss of ball velocity after impact.

	Feet							Diagram No. in "Billiards.." by Hoppe
Shot No.	S_1'	S_1	Measured S_2	Corrected S_2	Corrected as per	$\frac{S_1}{S_2}$	Notes on Cueing	
1	.942	.404 .612 .758	.583 .942 1.281	.619 1.030 1.450	#1	.653 .595 .523	Moderate stroke Right english, above center	62--the No. 3 Shot
2	.884	.600 .833	.816 1.290	.835 1.34	#2-7	.720 .621	Moderate stroke Running english A bank shot.	81
3	.358	.338 .645	.483 .984	.488 1.009	#2-7	.693 .640	Moderate stroke Left english	57, Right
4	.775	.283	.400	.401	#2-7	.706	Hard stroke Right english, ½ above center	60, Right.
5	.700	.367	.491	.499	#2-7	.735	Hard stroke Right english, ½ above center A difficult shot	Much as in 60, Right.

Table V. Eight Shots.

Measurements and corrections: as noted in Table IV.

Shot No	Feet S_1'	Feet S_1	Feet Measured S_2	Feet Corrected S_2	Corrected as per	$\frac{S_1}{S_2}$	Notes on Cueing	Diagram No. in "Billiards--" by Hoppe
6	.745	.381	.521	.530	#2-7	.720	Moderate hard stroke Slight left english; cued near center Cue butt slightly elevated	58, Right, essentially
7	1.166	.779	1.368	1.425	#2-7	.546	Moderate stroke Slight left english The Break Shot.	42
8	.925	.787	1.225	1.265	#2-7	.622	Moderate hard stroke Extreme left english. Long through stroke, sudden stop at end to avoid object ball	Difficult, unusual shot.
9	.942	.451	.60[illegible]	.610	#2-7	.740	Moderate stroke Ball cued close to center	
10	.963	.593	.834	.852	#2-7	.696	Hard stroke Right english, ½ below center Probably, h greater than 0.48	61, Right
11	1.066	.486	.637	.650	#2-7	.748	Moderate stroke Right english, near center	51
12	.545	.402	.670	.685	#2-7	.586	Easy stroke Above center ½ cue tip ordinary follow shot	10, the No. 1 shot
13	.694	.312	.510	.520	#2-7	.600	Easy stroke Right english Difficult, unusual shot.	

are actual distances. The direct measurements made on the tracings of the photographs, uncorrected, are not included in this paper.

Cue Average Velocity.

If the photographs had been taken at a known flash frequency, it would be a simple matter to compute the average velocity of the cue, from point of impact to wherever the flash catches the cue,- provided, however, that the cue was flashed exactly at impact. The proviso would, of course, be fulfilled only by rare accident. It then becomes necessary to use ball velocity in order to find cue velocity. But the ball velocity, which is v_2 immediately after impact, begins to drop off within the range needed for this part of the study.

In Shot 1, ball velocity dropped rapidly, as compared with the other shots. By means of a special study (not included here) it was found where the ball would have gone to, had it retained its initial velocity, in the same time. In Table IV, S_2 is given as measured (corrected for foreshortening) and also as corrected for decay of ball velocity. In Shots 2 and &, the rates of ball velocity decay were almost the same. These rates were averaged, and applied as corrections to all of the shots except Shot 1. Hereafter, in discussion, S_2 will refer to the finally corrected value.

If, in every case, v_2 were 1 foot per second, then the ratio S_1/S_2 would give the average cue velocity, from impact to where the cue is, in feet per second. But it is seen (in Section II) that for a given condition of impact (h fixed) the several velocities u_1, v_1 and v_2 are in fixed ratios to each other: if cue velocity is doubled just before impact (for instance) then the other velocities are doubled.

Therefore, for the purpose in mind, it is permissible to consider all cue ball velocities to have been 1 foot per second.

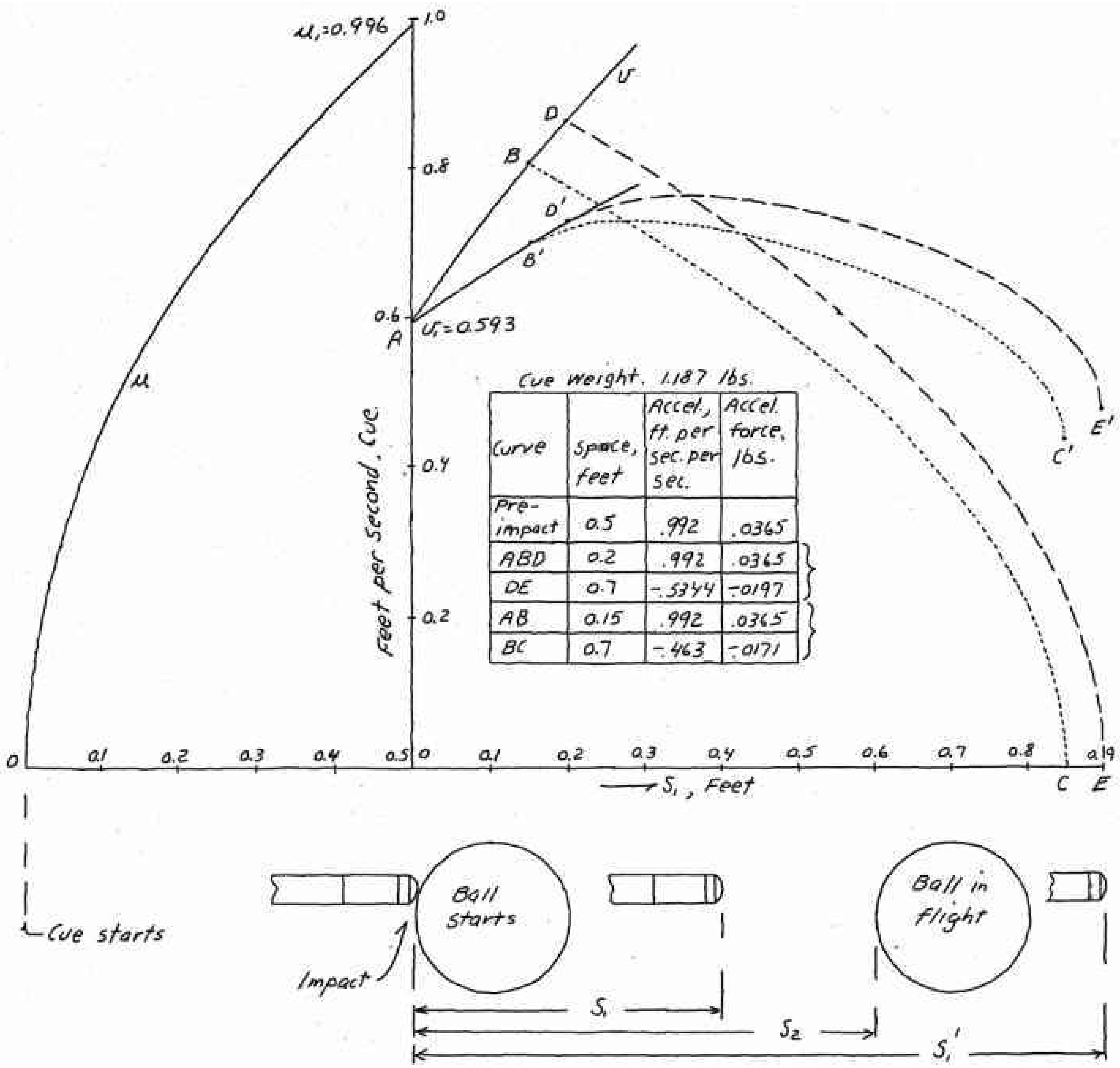
Shot 1 gives us three positions of ball and cue, and therefore three values of S_1/S_2. If these are plotted against S_1, a curve of cue average velocity results. This problem is then posed: what accelerations can be adopted for the cue, before and after impact, that will make the cue motion fit this curve and also fulfill all other requirements? The writer made three different kinds of approaches to this problem, and carried out perhaps 18 solutions in all. Only the final method, and the better-fitting solutions, are given here.

Theoretical Cue Behaviour.

For the time being, we now drop consideration of actual cue behaviour and turn to theoretical cue behaviour.

Take the case of a ball cued at h = 0.48 (Section II). If ball velocity v_2 is taken as 1.00, the pre-impact cue velocity u_1 is 0.996, and post-impact cue velocity v_1 is 0.593. These are marked on the vertical axis, Fig. 3.

A study of the "bridge" used (see "Billiards...") shows that for



Cue weight. 1.187 lbs.

Curve	Space, feet	Accel., ft. per sec. per sec.	Accel. force, lbs.
Pre-impact	0.5	.992	.0365
ABD	0.2	.992	.0365
DE	0.7	-.5344	-.0197
AB	0.15	.992	.0365
BC	0.7	-.463	-.0171

Cue Velocity and Cue Average Velocity
Theoretical Curves

Fig. 3

Shots 1 to 5, the cue typically is accelerated, before impact, through a distance of 6 inches, or 0.5 feet. It is a simple matter of mechanics (not included here) to work out the u-curve, cue velocity versus distance, Fig. 3. During acceleration, it changes from zero to 0.996.

During impact the cue loses energy, and cue velocity drops to 0.593. Then, if we assume that the same accelerating force continues to be applied for a while, the cue velocity rises along the v-curve, ABD... . Let the force be applied until D is reached, 0.2 feet after impact.

Suppose next, that a constant negative force is then applied to the cue, of such a value that it must stop the cue in a slow-down space of 0.7 feet. Again, the falling curve of velocity, DE, is easily worked out. The post-impact velocity curve for the cue is then ABDE.

With that done, data become available for finding the total post-impact time taken by the cue, up to any value of S_1. Divide S_1 by its total time, and we get the average cue velocity up to that point. The curve ADE' is the result. These curves were carefully computed, and are correct to within 1%.

Or again, if the post-impact accelerating force is continued for only 0.15 feet, then constant decelerating force for the next 0.7 feet to stop, the post-impact stroke totals 0.85 feet; the cue velocity curve is then ABC; and the corresponding curve of average cue velocity is AB'C'. Obviously, any number of assumptions like these could be made, thereby leading to any number of sets of curves like the two sets described. Our interest lies with such sets of curves as best fit all of the requirements.

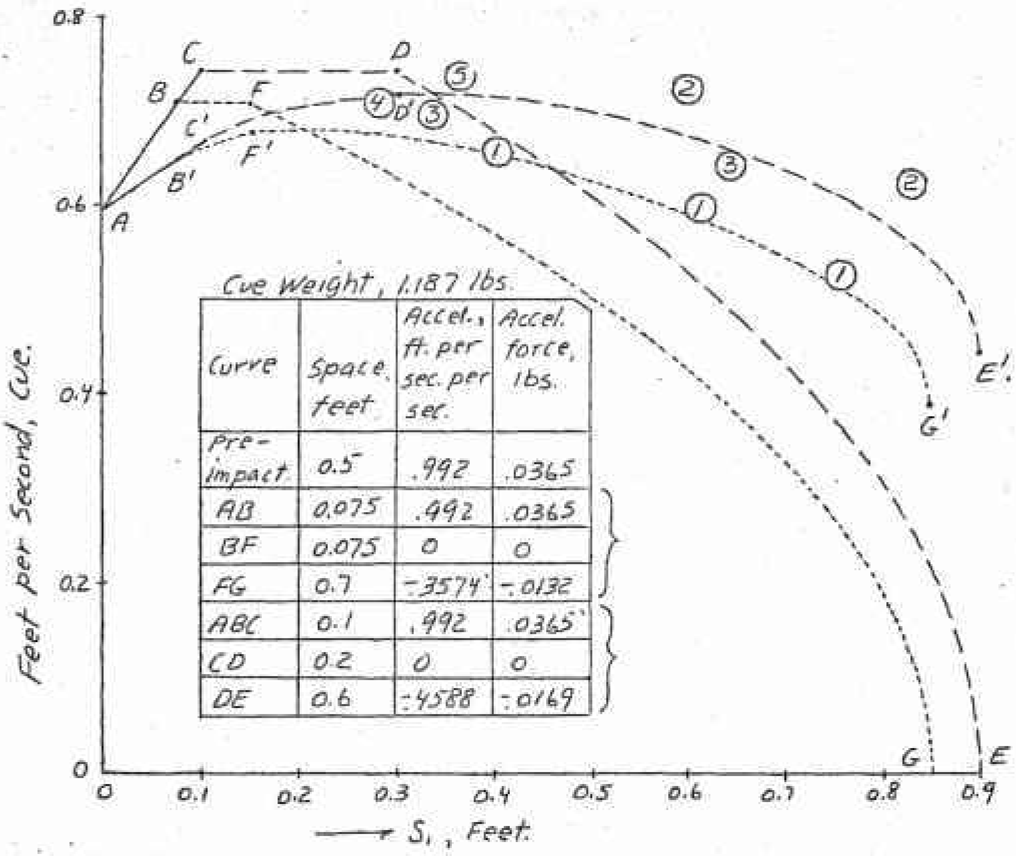
Strokes - Actual and Theoretical.

Turn next to Fig. 4, and first consider the average cue velocity curve described by the three points of Shot 1. The writer was quite unable to work up a theoretical curve to fit these points, and still stick to the idea that once acceleration ended, deceleration should at once begin. Thereupon, a new factor was put into the case: let the cue coast, without force applied, for a brief period after acceleration ends and before deceleration begins.

So, we will now let acceleration continue after impact for 0.075 feet; permit coasting for 0.075 feet; then decelerate in 0.7 feet: total post-impact stroke, 0.85 feet. Before impact, same conditions as first described above. It is seen that we get an excellent fit with the Shot 1 points. ABFG is the velocity curve, and AB'F'G' is the average velocity curve that fits this particular Hoppe stroke.

The reader can now interpret for himself the other set of curves, ABCDE and AB'C'D'E'; the accelerating hangover is a little longer, the total stroke is longer. The average velocity curve fits quite well with the two points of Shot 3, and is very close to the points for Shots 4 and 5.

Passing to Fig, 5, first note that the two points of Shot 2 have



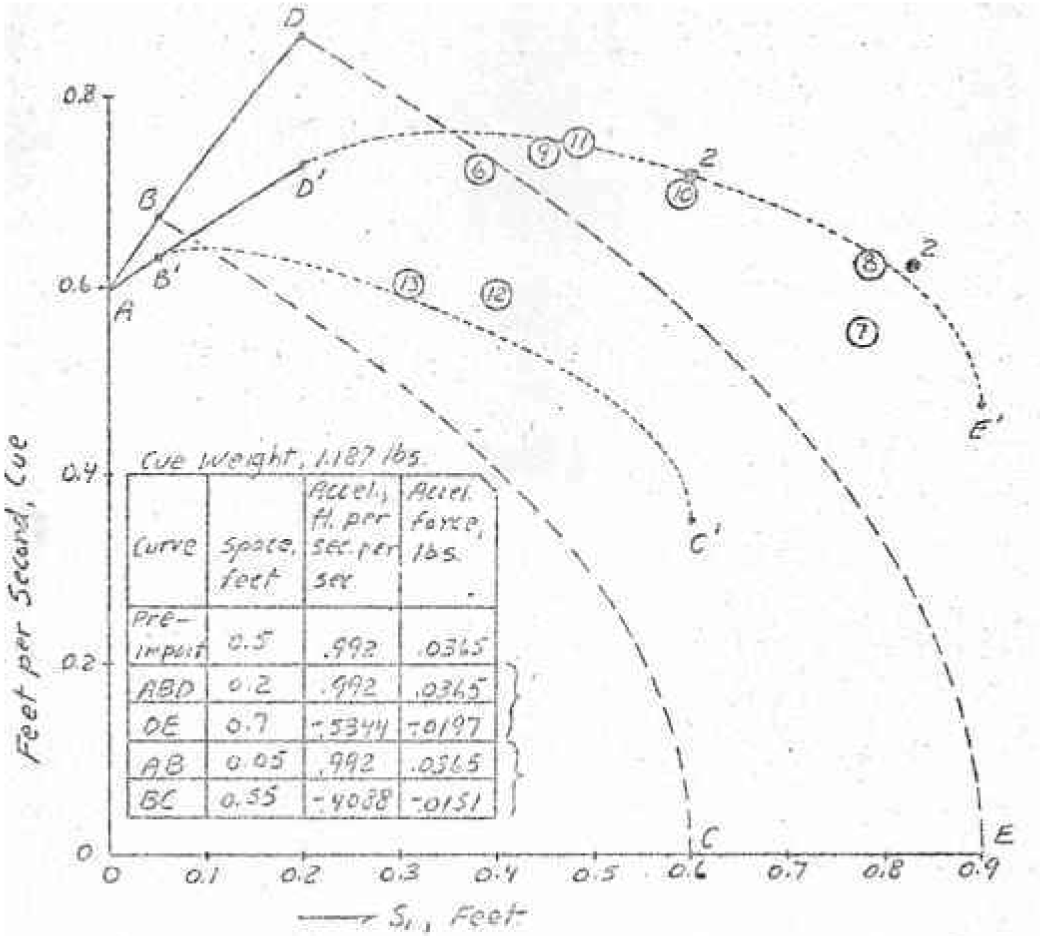
Cue Weight, 1.187 lbs.

Curve	Space. feet	Accel., ft. per sec. per sec.	Accel. force, lbs.
Pre-impact.	0.5	.992	.0365
AB	0.075	.492	.0365
BF	0.075	0	0
FG	0.7	-.3574	-.0132
ABC	0.1	.992	.0365
CD	0.2	0	0
DE	0.6	-.4588	-.0169

Cue Velocity and Cue Average Velocity, Theoretical Curves

Also, Hoppe Shots 1 to 5, Cue Average Velocity Points.

Fig. 4



Curve	Space, feet	Accel., ft. per sec. per sec.	Accel. force, lbs.
Pre-impact	0.5	.992	.0365
ABD	0.2	.992	.0365
DE	0.7	-.5344	-.0197
AB	0.05	.992	.0365
BC	0.55	-.4038	-.0151

Cue Velocity and Cue Average Velocity, Theoretical Curves

Also, Hoppe Shots 2, and 7 to 13, Average Velocity Points

Fig. 5

been included here again, and that a very good fit with them is obtained by leaving out the coasting space.

Also note that in spite of diversity of kinds of shots, ease or hardness of stroke, total stroke, etc., the points for Shots 6 to 11 fall closely along the upper curve of Fig. 5 or the upper curve of Fig. 4.

Short Strokes.

Shots 12 and 13 give low points, Fig. 5. But these are both short-stroke points. Note that the short-stroke theoretical curve in the lower set, Fig. 5, accounts well enough for these points.

DISCUSSION

The basic data are scant; and even though theoretical curves might be worked up to fit all of these "Hoppe points" exactly, it does not follow that they would be entirely correct. However, be it remembered that the points were fitted quite well; that the assumptions underlying the theoretical curves were simple rather than complex; and that the spaces covered by the stroke, both before and after impact, were essentially correct.

Constancy of Force.

Does Hoppe use a nearly constant accelerating force, prior to impact? The writer believes that he does, and believes so for two reasons. First, the expert adopts techniques that are simple, reliable, and such as can be accurately learned and executed. It would then seem that a constant force would be adopted by the expert, even though the adoption be unconscious. Second, the writer has made a study of his own reactions; and as nearly as he can tell, he favors constancy of force.

Acceleration after Impact.

Next, does Hoppe maintain, for a while after impact, the same accelerating force he uses up to impact? The answer is that he cannot do anything else, and achieve reliability. The pre-impact stroke stroke must furnish the right cue velocity at impacts Any attempt to cut off the force immediately after impact would sometimes cause it to be cut off before impact. And here, perhaps, lies the main secret in the "follow-through" stroke. In all games of skill requiring a stroke, the professionals teach the necessity of having a follow-through. The necessity for having at least a <u>brief</u> follow-through has been made clear above.

Coasting Period.

As to the coasting period, the necessity is not clear. From the evidence, it appears that Hoppe usually inserts a coasting period, when all is clear ahead and there is no reason to cut off the stroke abruptly.

Long Follow-Through.

This brings us to the surprisingly long and apparently unnecessary decelerating part of the stroke: it ranges from 0.7 feet to well over a foot. A good player can use a very short post-impact stroke and yet put plenty of energy into the ball, when he has to. But on the majority of shots, when there is plenty of room ahead and he is free to use the long follow-through, he uses it. Why? This amazingly long stroke - which has not the slightest effect on the ball - cannot be waved aside as a mannerism, a widespread custom, or useless tradition.

Two tentative explanations of why it is a real factor in the game will be offered. First, an expert is a man who does one thing at a time when the technique permits - and does it very well. By not having to worry about giving the cue a quick stop, Hoppe is able to concentrate on the one important thing: cueing the ball correctly. The second proposed explanation grows out of watching the "body english" that many amateurs exhibit: after cueing the ball, they swing the cue and the body in a cheer-the-boys-on manner, Psychologically, "body english" is a wholehearted gesturing amounting to a prayer for success. And the professional? Have we any right to think that he is free from having the same desires and emotions? Of course not. And if not, then it may be that his long post-impact stroke is his dignified brand of body english - polished, perfected, and unconsciously used as an urging-on and a prayer for success.

Facts the Expert Cannot Supply.

It is idle to ask Hoppe to describe the forces he applies to the cue. He does not know. His right forearm and wrist and hand have long since been developed into several pounds of habits,- several pounds of bone and muscle that do the job automatically. In response to the question as put by the writer, Mr. Hoppe said that in play, he hardly knows he has a right arm: it feels dead to him,- almost as if it were paralyzed.

Decelerating Force.

Does Hoppe use a constant decelerating force? The evidence is that the force is reasonably constant, at least over a large part of the slow-down space. Near the very end of the stroke, Hoppe usually closes his hand around the cue; and he uses this late-in-the-stroke grip to stop the cue when an abrupt stop is in order. Until then, his grip consists only of snugly encircling the cue by thumb and forefinger only.

Pendulum Swing versus Side-Arm Stroke.

The average amateur's play discloses many faults, One of them is in failing to swing the forearm vertically from the elbow, with the elbow fixed in space. The cue is not then driven straight ahead at the ball. Now, in terms of pure mechanics, it is not at all necessary for the cue to travel straight ahead. If its line of action is correct at impact, if it cues the ball at the right spot, and if the velocity is correct, the shot will be made. Yet it is true that the amateur seldom learns to play well until his stroke goes straight ahead, Yet it is also true that a few amateurs master a stroke having a side-swing to the cue, and do very well with it.

In "Billiards...", Hoppe strongly insists that everyone's stroke should go straight ahead, even to the end of the stroke. It is therefore most interesting to find from the photographs, that Hoppe himself never makes a long stroke of that kind. He cannot make it. He is a "side-wheeler", a "sidewinder", a side-arm player. As a boy, playing a good game at age five, he had to raise the arm out sidewise in order to play at all. He still raises the arm. His forearm does not hang vertically from the elbow. With such a swing, it is impossible to achieve a straight-through stroke. Every photograph that yields any evidence shows that Hoppe's cue tip wanders definitely to the right in the post-impact travel. Hoppe told the writer that he believes he himself should use the pendulum swing, and would probably by now have changed over — but for the time it would take to re-learn. He mentioned one professional who did change over. It took him eight years to do so.

Many have ascribed a part of Hoppe's success to his unorthodox swing. Hoppe does not agree. But the writer feels that when the world's greatest billiard master uses a side-arm swing, a fact is presented that should not lightly be passed by. There is just the possibility that the advantages of the straight-through stroke should be sacrificed, in favor of the possibly greater advantages of a side-arm swing. We cannot forget that Hoppe's right arm handles the cue in much the same way that a violinist handles his bow. And again, a dual possibility may be the answer. Perhaps it is that to become a good amateur, reliable cueing can only be attained by using the pendulum swing; and perhaps, it takes a master player to be able to master the more uncertain side-arm swing and thus really wring from it the advantage of greater delicacy of touch and control.

Conclusions.

The only way in which Hoppe has complicated his technique, is by retaining his side-arm swing. By retaining it, he has avoided the possibly greater complications to be encountered over a period of years if he changed over to the pendulum swing. Also, by retaining it, he is almost certainly achieving greater delicacy of stroke.

Otherwise, every element of Hoppe's stroke is of the simplest possible character: application of a constant or near-constant force before, during, and for a period after impact; a coasting period usually introduced; and a long slow-down with constant or near-constant force,- a perfectly natural running-down process that requires no thought and which therefore does not, by virtue of having to anticipate it, get in the way of executing properly the pre-impact acceleration.

Simplicity is further achieved by adjusting the pre-impact space to get a change in cue velocity. Hoppe could use a fixed bridge distance, and vary his accelerating force; and on occasion, he does vary the force when it has to be done. But within the limits offered by good play, he varies the bridge distance and achieves simplicity by using the same force applied over longer distances.

SECTION IV. THE COURSE OF THE BALL: ANALYSIS OF SOME OF THE PHENOMENA.

The expert player makes the cue ball do some things that fascinate and mystify the observer. They also fascinate and mystify the expert. As in every sport, the expert masters the phenomena, and knows how to produce them, even though he does not understand them. The purpose of this part of the paper is to take the mystery cut of some of the major phenomena exhibited during the complete course of the ball. As far as is known, this is the first time that acme of the ball's queer behaviourisms have been explained.

The Break Shot.

Fig. 6 shows the "break" shot. It is Shot 7 of the photographic studies of Section III. Every game starts with the break shot. The writer has worked out the ball velocities along the course of the cue ball and of the first object ball.

In order to work out these values, it was necessary to know the total time taken by some one ball during its roll from one cushion to the next. To establish times for several parts of these courses, the writer repeatedly made parts of the break shot, and timed the paths with a stop-watch. Among the numerous trials, some were nearly enough duplicates of the Hoppe break shot (when velocities were such that the balls rolled to about where they did for the Hoppe photograph) to warrant the times taken on them.

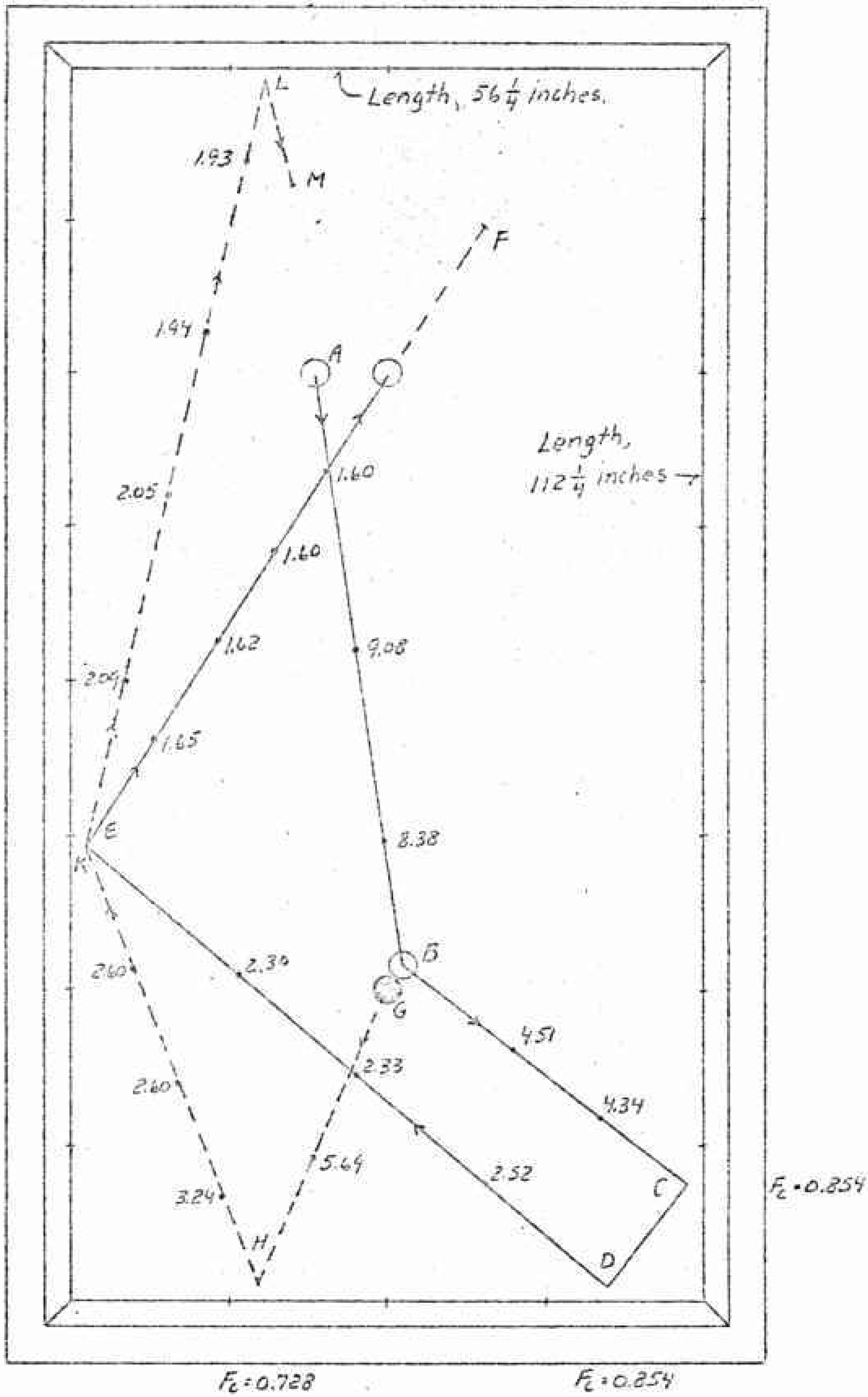
Ball Velocities.

This timing was done on the large table at the University Club, only a few weeks after Hoppe made exhibition shots en that table. At the time, he included (and made) his famous nine-cushion shot. It is concluded that this table was in good condition. It is further believed that the ball velocities (the numbers spotted along the course, Fig. 6) are correct to within 5%. As compared with each other, they are for the most part correct to within about 2%. That is, they may all be slightly high, or all slightly low. The velocities are in feet per second.

The velocities are plotted along the entire course of the cue ball in Fig. 7. In Fig. 7 (which is lettered to correspond to Fig. 6) note the short marks across the line AB at P and Q. These marks mean that along path AB, the photographic flashes caught pictures of the cue ball at P and Q. The real distance from P to Q, along with detailed computations not included herein, enabled the P-Q velocity of 9.08 to be found. This velocity was arbitrarily plotted at the middle of the PQ span. This procedure is not strictly correct, but it is accurate enough for these studies. The QB span gives the next velocity, amounting to 8.38 feet per second.

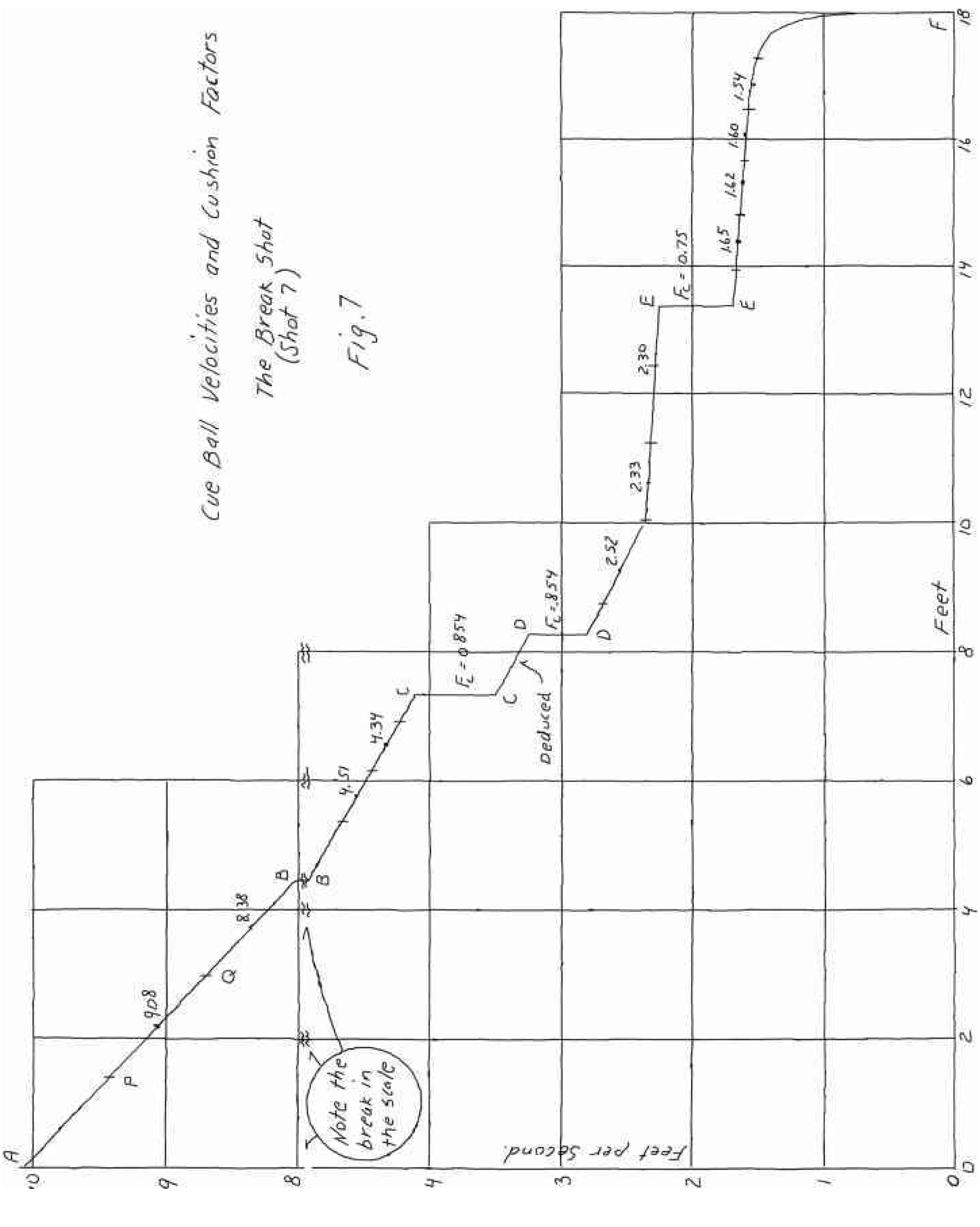
The sharp vertical breaks in the curve occur when the ball strikes the cushions. The end of the course, F, is the point to which the ball would have rolled, had it not encountered the second object ball.

The path DE at once stands out as peculiar. For about two feet;



The Break Shot

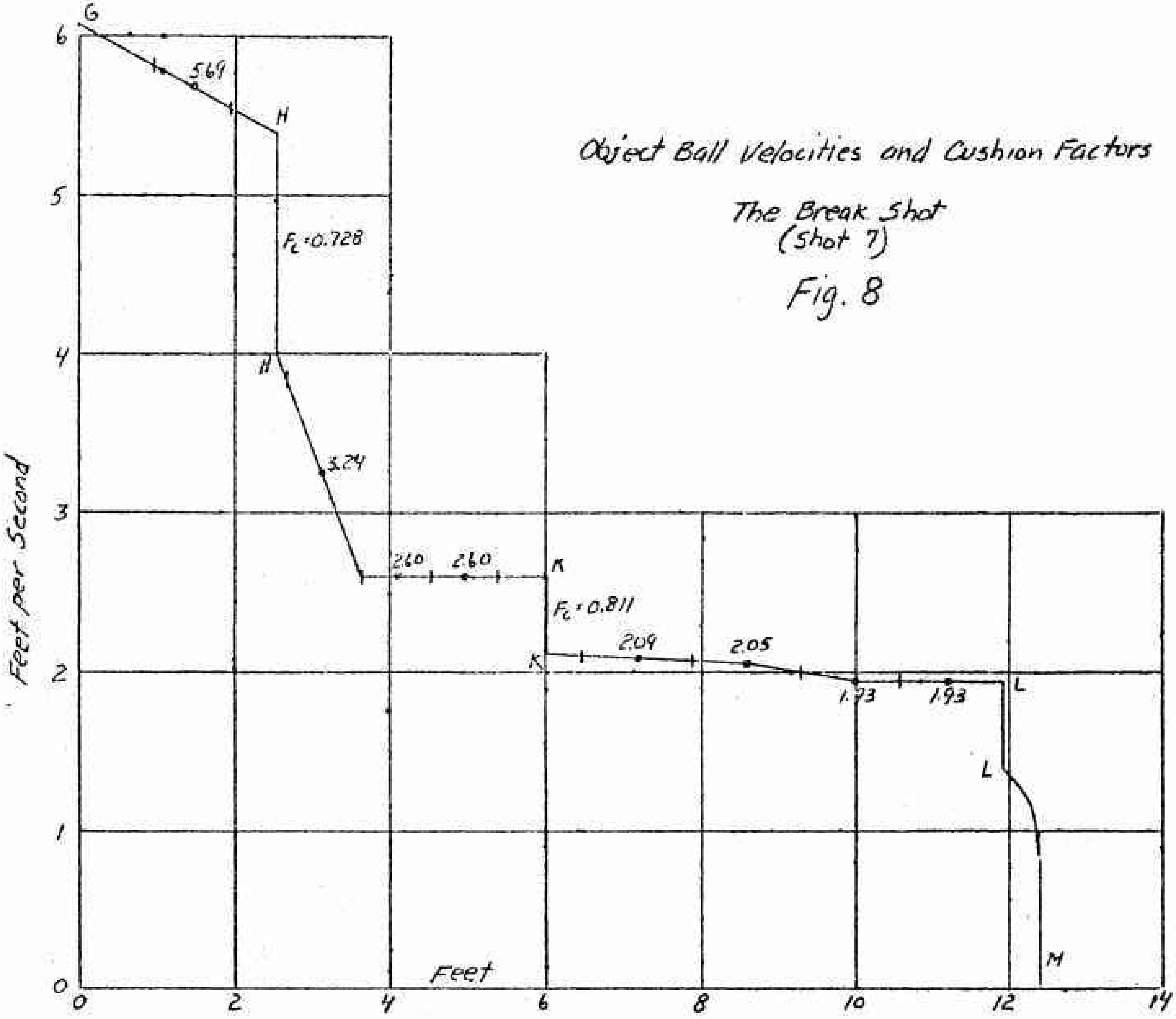
Fig. 6



Cue Ball Velocities and Cushion Factors

The Break Shot (Shot 7)

Fig. 7



Object Ball Velocities and Cushion Factors

The Break Shot
(Shot 7)

Fig. 8

the ball rapidly loses velocity; for the next three feet or so, its velocity is nearly constant. The reasons for such behaviour will be found most interesting.

Pass to Fig. 8, which shows velocities along the course of the object ball; we note the same peculiarity showing up in path HK.

Cushion Factor.

We now interrupt the study of the ball inflight, to note what happens when it strikes a cushion. The velocity curves enable us to find the velocities obtaining just before and just after the impact with the cushion. A "cushion factor" has been adopted, and given the symbol F_C. F_C is the ratio of the two velocities. The cushion factors have been noted on the breaks in the velocity curves in Figs. 7 and 8. The two charts include five cushion factors in all. The lowest is 0.728 and the highest is 0.854. The factor, as defined and computed, takes only translational velocities into account, and ignores ball spin (rotational velocity). It cannot then be used to compute "cushion efficiency", but it does give a fair indication of how much or little of energy is lost at the cushion.

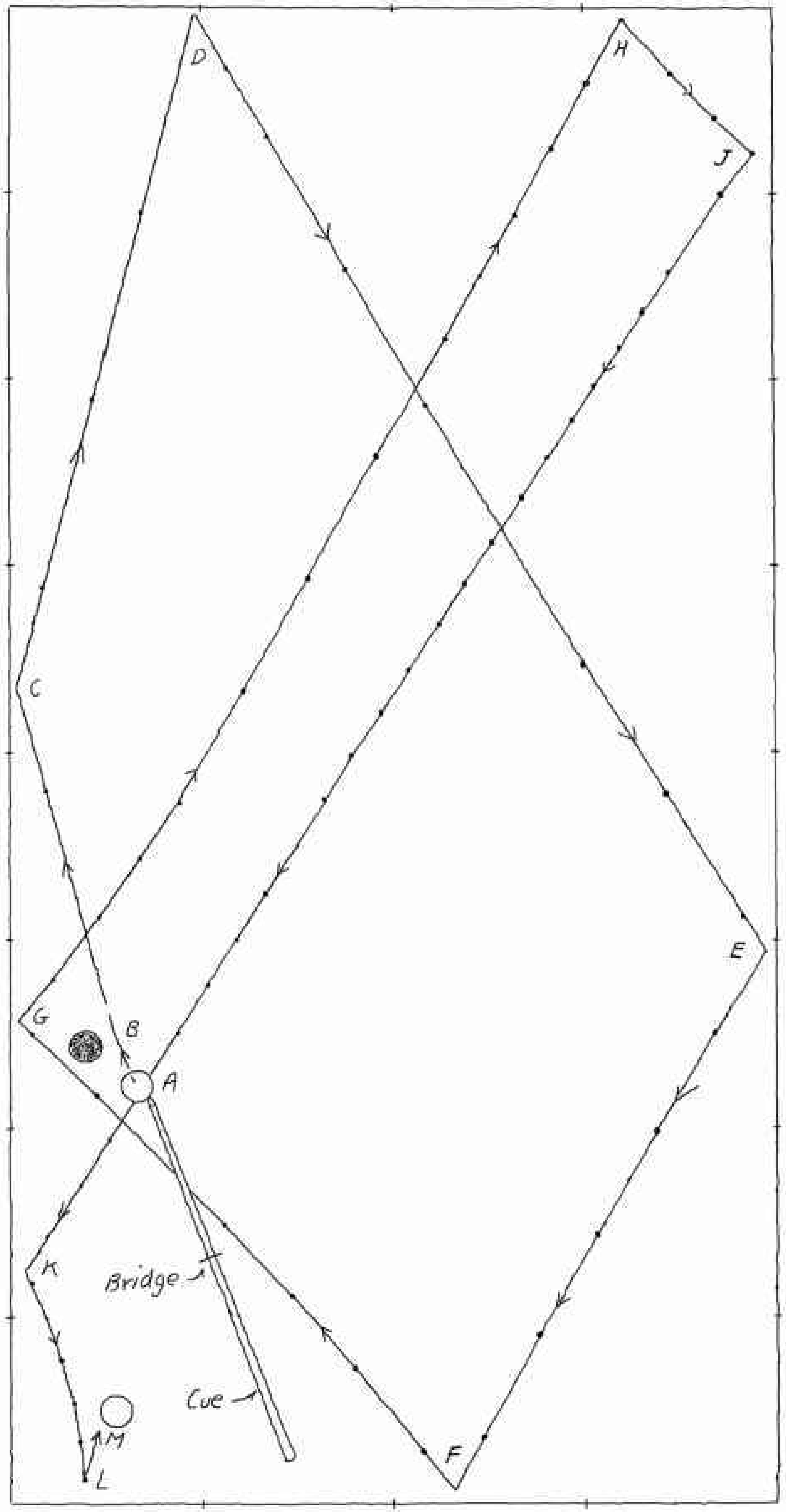
The Nine-Cushion Shot.

Passing to the nine—cushion shot of Fig, 9, the opening remark should be that it is amazing that anyone can make a ball go far enough, hitting so many cushions, as to make such a shot at all. If the fact itself is surprising, the explanation is even more so. Everyone seeing this shot or hearing of it, naturally assumes that the cue ball must start off with very high velocity. The writer himself started out with that assumption. The assumption is contrary to fact. This is a real surprise. In fact, when the writer first extracted from Fig. 9 some of the facts about the ball's behaviour, he did not believe them: he started looking for mistakes in the work, and even suspected that for once, Gjon Mili's flash timing mechanism had been acting up and had allowed the flash intervals to vary. But the findings turned out to be true, nonetheless.

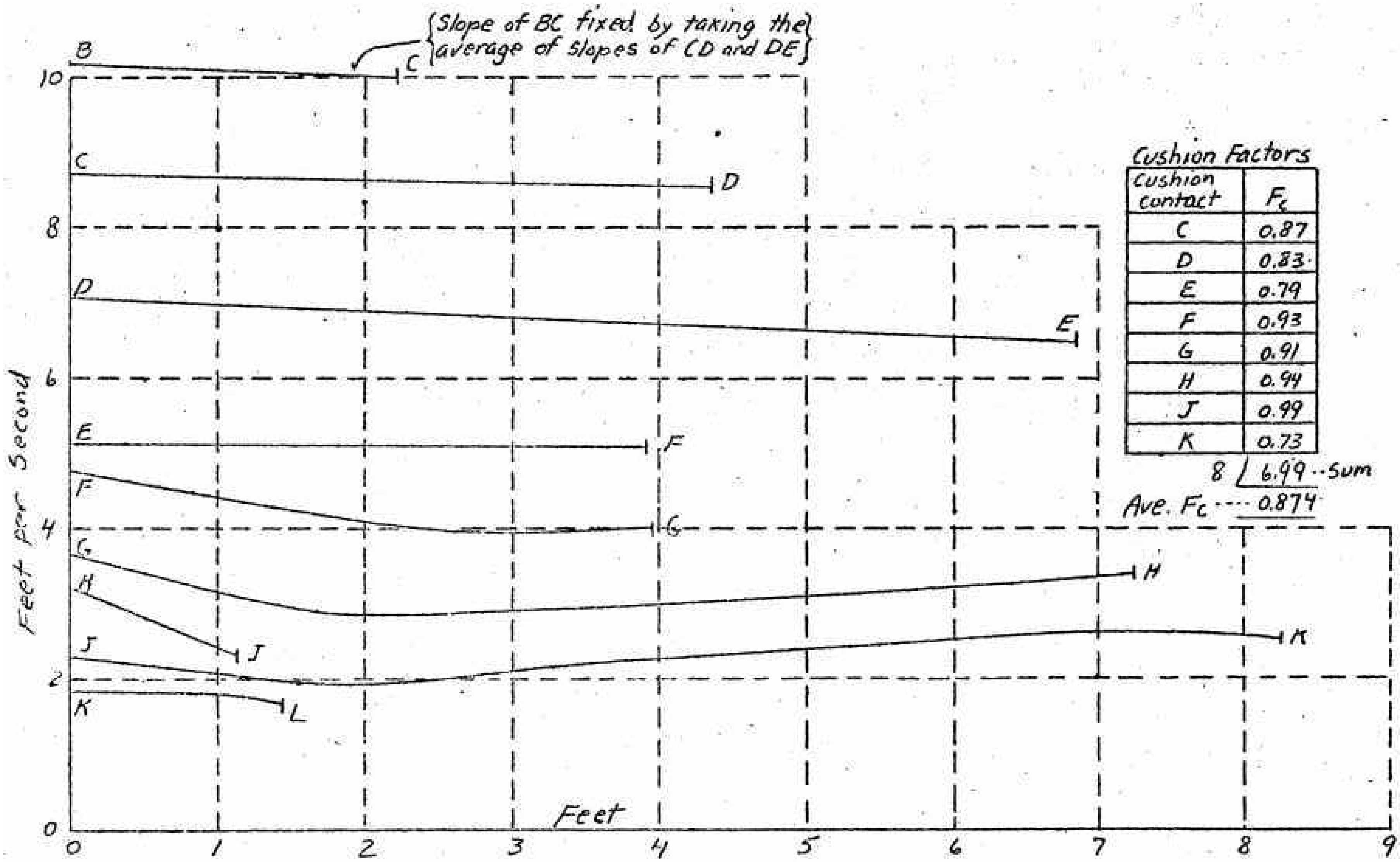
In order to work out the cue ball velocity curve, a velocity some¬where along the course had to be known. To get an approximation, we may observe that in the break shot, Fig. 6, on the path KL, the ball approached L at about 3 feet per second. It was therefore assumed that on path HK of Fig. 9, the ball approaches K at somewhat more than 3 feet per second. There will be some error thus introduced, but it is not large. Then, the cue ball velocity curves of Fig. 10 were worked out. As compared against each other, these velocities are about right. They all may be a little high or all a little low.

Discoveries.

Three startling discoveries demand attention. First, the cushion factors (see Fig. 10) average 0.874, and the one at J is nearly unity. Second, the initial cue ball velocity at A (which is nearly the same as when the ball was cued) is only about 10 feet per seoond,- no more than Hoppe used for the break shot! Third, after the ball leaves cushion contact C and starts on its "natural" flight, certain



The Nine-Cushion Shot
Ball Positions and Course as Photographed
Fig. 9



Cushion Factors

Cushion contact	F_c
C	0.87
D	0.83
E	0.79
F	0.93
G	0.91
H	0.94
J	0.99
K	0.73

8 ⟌ 6.99 ..Sum

Ave. F_c ···· 0.874

Cue Ball Velocities and Cushion Factors

The Nine-Cushion Shot

Fig. 10

path velocity curves show a drop in velocity, following by a rise.

In billiard terminology, such a flight around the table is called a "natural". Whoever gave it that name made a fine choice of terms. The remarkable self-sustaining qualities of the ball flight will be investigated below.

Path Curvature.

One more characteristic, seen in Fig. 9, must be mentioned: somewhere along a path from cushion to cushion, shortly before the velocity begins to rise, there is a curve in the path. Looking along any one of these paths (looking the way the ball goes) the ball swerves to the left. The two parts of the path before and after curving are often straight or nearly so. The path deflection is typically from one to several degrees, in ordinary shots. (Note: of course, if the shot is reversed and made to go naturally around the table the other way, the path will swerve to the right.)

Opposite Path Curvatures.

From billiard experience, the writer had long known that when a ball goes from cushion to cushion as in Fig. 9 and is therefore spinning counter-clockwise, it sometimes curves to the left. But it was disturbing to possess another bit of knowledge: if a ball is cued with right english, thereby making it spin counter-clockwise as did the previous ball, it curves to the right. Two balls: both spinning counter-clockwise; one leaving a cushion and curving left; the other leaving a cue and curving right. A dilemma.

A typical test of the cued ball, made by the writer, may be cited. Cue the ball at h = 0.48, right side, and at about 3 feet per second: the ball's path will drift rightward by 2 inches in 6 feet.

In trying to reconcile the opposite behaviours of the two balls, the writer wasted many experimental shots, in trying to cue a ball as above and make it curve to the left. Every effort failed. But if "cushion" english would do it, why not cue english? The writer had to extend his observations before he was able to start on the right lines of analysis.

The observations were made by starting a ball, then trotting alongside the table to observe the position of the spin axis. At ordinary velocities, the spin axis can be quite well observed. The behaviour is brought out in Fig. 11.

The Ball Cued with English.

Consider the cued ball, Fig. 11. If cued with right english at the belt line, it would be given a forward velocity; and, if it were a free body, it would spin about a vertical axis. As far as the human eye can tell, the ball almost at once selects the axis shown at (1), instead of the vertical axis.

Let us adopt the terms tilt and lean, in describing the departure of a spin axis from the vertical. Observation shows that in the first few inches after the ball is cued, the axis is tilted across the path about 45 degrees, and also leaned ahead by perhaps 10 degrees.

Ball behaviour now begins to clear up, and it does so in terms of such matters as table friction, action of the ball on the table, transfer of energy from rotational to translational form or the reverse, and gyroscopic action.

The Ball as a Gyroscope.

When the billiard ball spins, it is a gyroscope: its spin axis tends to remain fixed on the same distant point in space. Next, if a force is applied that tends to shift the axis, two things happen: the force is resisted, and there is precession of the axis. Precession means that instead of the axis shifting the way the force tries to shift it, it shifts instead at right angles to the force tendency and continues to do so as long as the shift force is applied.

Another description of precession is to say that if the force were applied long enough, the axis would shift so that the spin would be the kind of spin the applied force itself would produce if it could.

Return, then to cueing the ball, Fig. 11. Exactly what happens during impact and immediately after, we cannot know, for undetermined table friction enters in. But apparently, friction is high at the outset, as friction often is in getting something started. The friction force tends to shift the bottom of the temporarily vertical axis to the rear. Precession occurs, and tilts the axis across the path about 45 degrees. But along with that, a second effect takes place. As the successive bottom points of the ball are skidded to the right against table friction during that tilt, a secondary crosswise force acts on the ball bottom, and this causes a second precession: it leans the axis ahead.

It must be presumed that now, with the ball well started, table friction drops to a normal value. The backward force of friction is reduced, and further tilt of axis occurs slowly. The ball is now at (1), perhaps an inch from where cued. As it moves along to positions (2, 3, 4, 5) - a matter of from 1 to 4 feet in many shots - some other things happen for which we turn to the corresponding views of Fig. 12.

Analysis of Forces.

In Fig. 12, we look down through the ball and see certain velocities and forces laid out on the table. These vectors are all concerned with the point of contact.

At position (l), the vector R represents the spin velocity of the ball at point of contact. Vector T is the translational velocity of the ball. The vector sum, V, is the net velocity of the ball's point of contact, with respect to the table. The frictional force acting on the ball is vector W, opposite to V.

The frictional force W has several effects. Be it noted that this force arises because the ball contact is sliding on the table, and this in turn derives from the fact that two velocities T and R are not matched. Note further that T and R could be made to match as to amount (if not as to direction) by carrying out any one or all of three processes: first, slow down the ball, thereby reducing T; second, make the ball spin faster, thereby increasing R; third, let the ball keep the same spin (same revolutions per second) but give more tilt to the axis, thereby laying a larger and faster spin circle of the ball surface onto the table.

The effect of force W is to carry out all three processes at once. It slows down the ball; it causes it to spin faster; and by trying to shift the axis, causes gyroscopic precession ... axis tilt across the path of flight is increased, and a larger spin circle of the ball is placed at the point of contact. Of course, the force W also accounts for some frictional loss of energy. In the foregoing process, some translational energy is converted into rotational energy.

Phase X. This slowing-down, spin-increasing, axis-tilting behaviour occurs in what we will call the Phase X period. As the ball continues in Phase X, it reaches (3) in Fig. 12. as shown in (2), T has been reduced, R has increased, the axis has been further tilted, V is reduced, and the force W has swung somewhat more crosswise of the path, The ball at (3), Fig. 11 and 12, is at the end of Phase X. Velocities T and R are now matched in amount, but they are out of line. Velocity V is, like W, now crosswise of the path.

Phase Y: Path Curvature.

The ball now enters Phase Y, wherein its path is curved to the right. The main force acting on the ball no longer retards the for¬ward velocity; instead, W is crosswise. It acts to accelerate the ball to the right, and curve the path, In fact, a rightward compon¬ent of W existed all through Phase X, and a small acceleration to the right must already have been built up in Phase X.

When the ball ends Phase Y, it is at the and of the more obviously curved portion of the path; it will be clear that this is about when force W disappears,- which is when V disappears. But V disappears when R and T are in line. But that, in turn, is when the axis is finally tilted directly across the path, without lean. See (4).

There is a secondary effect in Phase Y now to be mentioned. The force W, being crosswise, attempts to increase tilt of the axis; precession ensues, resulting in a slight decrease of the forward lean of the axis. This causes the cued ball to have less curvature in Phase Y than would otherwise occur.

Phase Z.

At the end of path curvature, Phase Z follows. Since the ball was given some rightward velocity across the path in Phase Y, it will tend to continue to move rightward; the ball will attempt to execute a slight component roll to the right in Phase Z. To whatever extent this happens, it places a smaller spin circle onto the table, making R less than T, The main effect is to slow down the ball slightly, let it pursue a nearly straight path, and let it maintain the axis tilt and lean of position (4) until the ball hits a cushion.

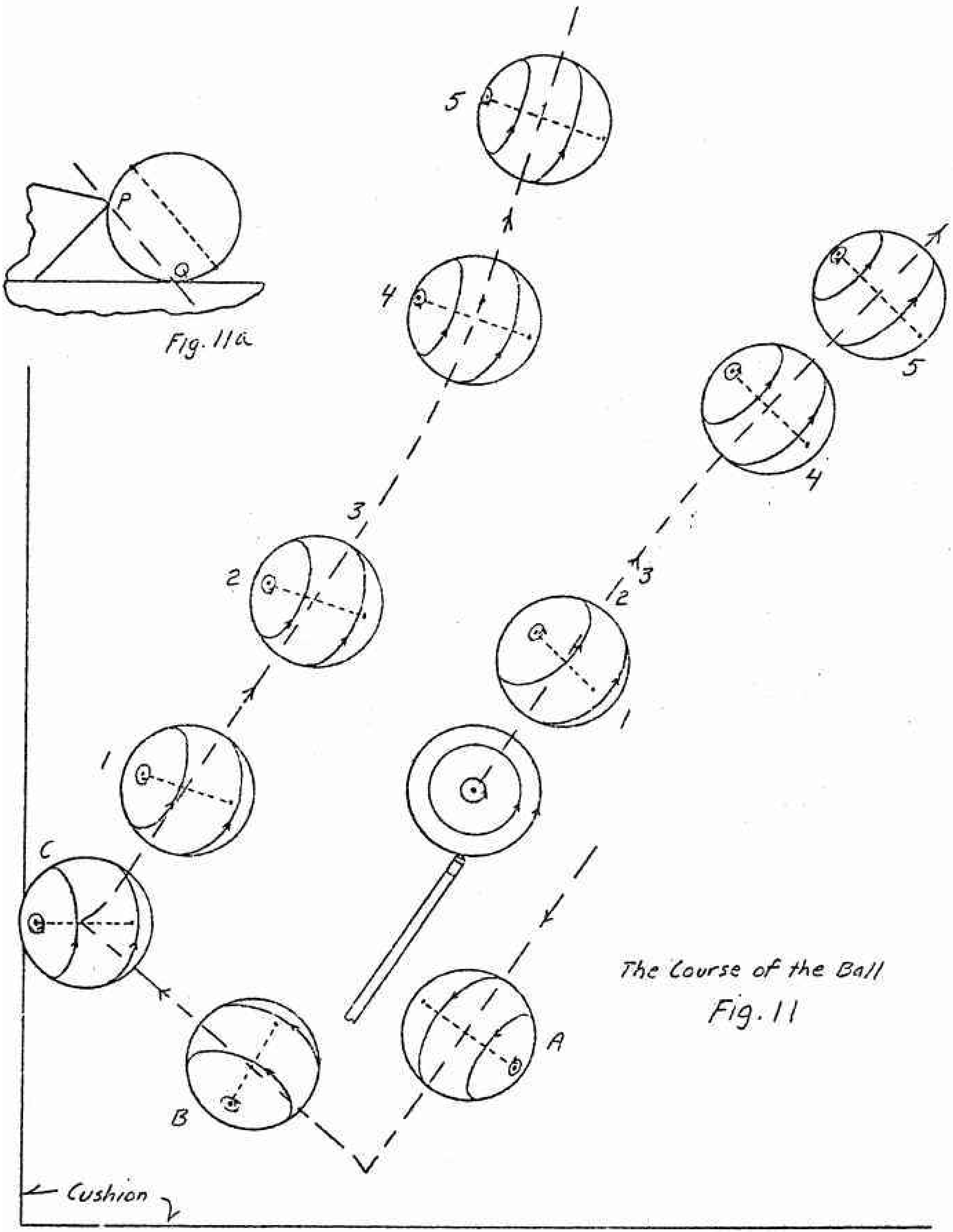


Fig. 11a

The Course of the Ball

Fig. 11

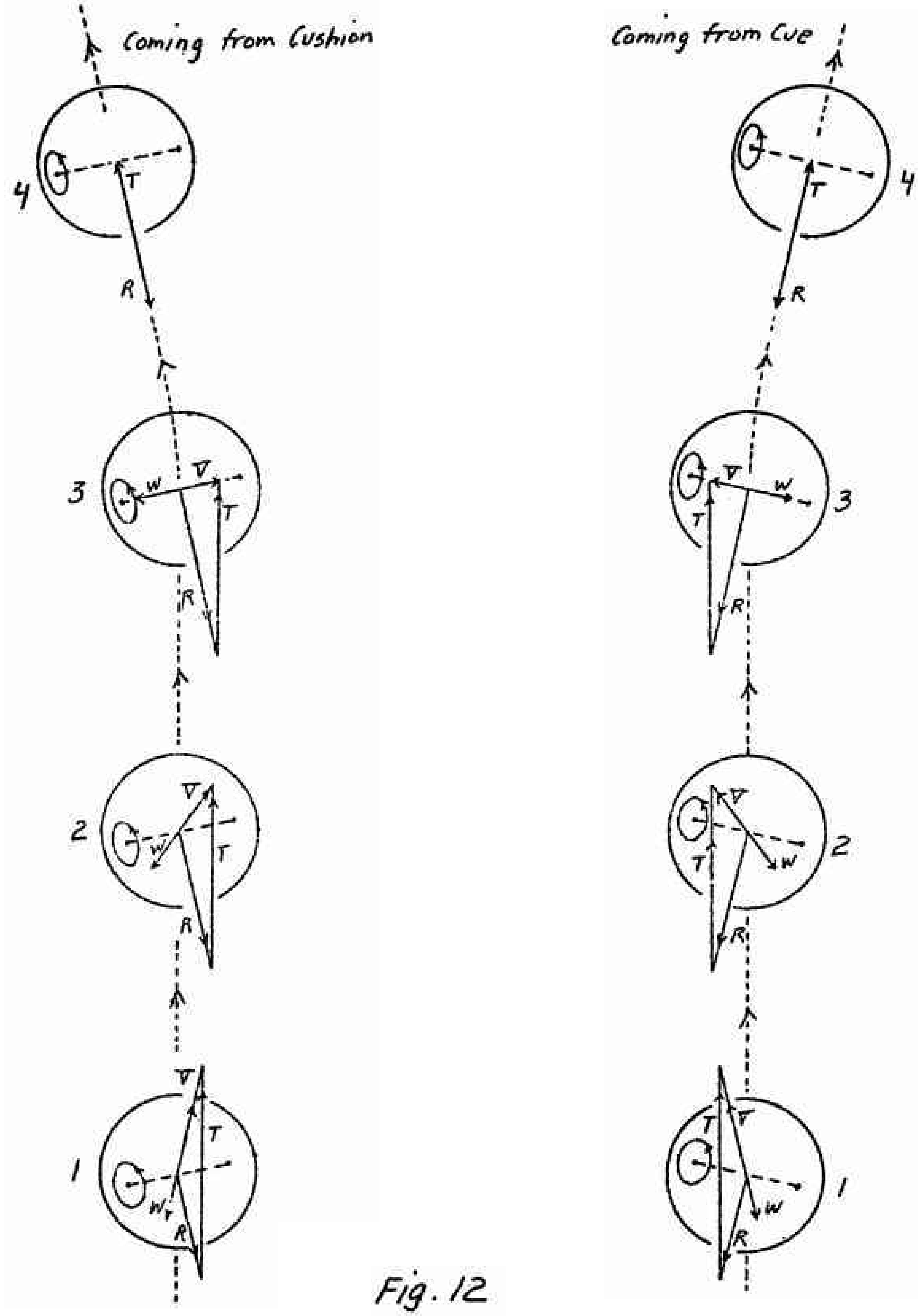

Coming from Cushion
Coming from Cue
4
T
R
3
W
V
T
R
2
V
W
T
R
1
V
T
W_r
R
W

Fig. 12

In the light of the foregoing, it seems clear that Hoppe's explanation (in "Billiards..") of the curvature of the cued ball's path, based on a simple frictional argument, should be abandoned.

Ball Leaving a Cushion.

In analyzing the behaviour of a ball leaving a cushion, we will find effects similar to those just studied for the cued ball, and also some effects that are different.

In Fig, 11, the ball is in natural flight, going from one cushion to the next, through positions ABC12345.

The ball at position (A), having left a cushion, approaches the next cushion with a spin axis about as shown,- provided its path length from the last cushion is fairly long and the velocity is not high. On its next short path, position (B) shows the condition of spin axis. The spin axes for the next long path are shown at positions (1, 2, 4, 5). These axes are all drawn in terms of the writer's repeated observations, made within the general velocity range of from 1 to 3 or 4 feet per second. The reader will at once note that these axes lean backwards, whereas the axis of the cued ball leans ahead. Therein lies the secret of the opposite path curvatures.

Action at the Cushion.

As to cushion effects, two things are certain, First, from the very high cushion factors (see Table, Fig. 10) we can be sure the cushion is a high-efficiency device. Second, the cushion rapidly and violently acts as an energy converter. As the ball pushes into the cushion, some or most of its energy may be stored in the cushion as elastic energy of deformation; but the cushion redirects and re-spins the ball and gives back most of the energy, as it pushes out.

The unaided eye is not fast enough to observe the action at the cushion. Therefore, the writer's speculations are offered for what they may be worth. When the ball has reached the point of greatest deformation of the cushion, the instantaneous axis would seem to be as shown in position (C), Fig. 11. The reason for thinking so (see Fig. 11a) is that then, the ball is gripped at points P and Q by cushion and table. Its real axis of rotation is then PQ. Its component motions would then be a translational motion along the cushion, plus a spin about the spin axis parallel to PQ.

As the ball is pushed out by the cushion, its center follows a short curved path, shown at (C). In springing back, the cushion's last act is to attempt to reduce the tilt of the axis. Gyroscopic effect would cause a rapid precession, causing the axis to lean ahead of the position it had at (C), Position (1) shows the axis leaning more ahead than it did at the cushion, but still leaning behind as far as the path is concerned. Observation shows that at (1), the tilt is about 30 degrees, and the lean backwards is perhaps 10 degrees.

Velocity Increase in Phase Z.

Along the first straight part of the path, positions (l, 2) the ball is in Phase X (see corresponding diagrams, Fig, 13), wherein the processes are the same as for the cued ball: loss of velocity, increase of spin, axis precessing to tilt further, until velocities R and T are equal but not in line. The ball then passes through Phase Y, with force W accelerating the ball across its own path and curving the path. Position (4) is at the end of Phase Y.

And now, a new phenomenon: the leftward crosswise velocity gained in Phase Y will cause the ball to continue to have a slight roll to the left. This will continually present larger and larger spin circles to the table. Velocity R continually attempts to grow larger than velocity T. This means that the ball's rotational energy is driving the ball, furnishing the losses, and <u>increasing its forward velocity</u> T. This is Phase Z of the ball-from-cushion path.

The phenomena of Phase Z sound much like getting something for nothing, but that is not the case. It is true that forward velocity T can and does increase in Phase Z, and that translational energy is increasing. However - because of the fact that axis tilt is increasing and laying larger spin circles on the table - the spin revolutions per second and the rotational energy are decreasing, A simple numerical solution carried out by the writer shows that all this can happen, along with the furnishing of some energy for losses, within such a range of tilt as from 40 to 65 degrees. As a check, visual observation shows that the axis does tilt through approximately this range in Phase Z.

There is a secondary effect to be mentioned. In Phase Y, the force W, in addition to moving the ball across the path and curving the path, tends to shift the axis gyroscopically by increasing the tilt. Precession occurs, and the net result is to increase the lean of the axis backwards. The effect of this is to increase still further the curvature of the path; whereas, the tendency of this secondary effect in Phase Z of the cued ball was to reduce the curvature.

Return now to Figs. 9 and 10, where some interpretations of behaviour can be made in terms of the above analyses. The velocity curves EF, FG, GH, JK, in the range of from 5 to 2 feet per second, all exhibit Phases X, Y, and Z; the slow-down, the flat-velocity region, and the speed-up region. Observe, Fig. 9, that these paths are distinctly curved.

<u>Fourth Phase: Rolling to Stop</u>.

The last long path is JK. The speed-up region, Phase Z, covers the remarkable range from the second to the seventh foot, or about 5 feet. Then the ball begins to lose forward velocity. It then enters a fourth phase: the axis tilt has become so great (about 70 or 75 degrees) that, even though further tilt due to cross-wise roll of the ball does lay larger spin circles on the table, they are not enough larger to have much effect. It is then (unless a cushion is encountered) that the axis lazily tilts over to become parallel to the table, and the ball makes a simple roll to standstill.

Phase Overlap; other Complications.

In the above descriptions of the three Phases, they were sharply distinguished to keep the analysis clear. But it must be recognized that Phase X overlaps Phase Y, and Phase Y may overlap Phase Z.

Another complication enters: change of gyroscopic effects along with change of spin velocity. To have a good gyroscope, rotational speed or spin should be very high. In the earlier paths of the nine-cushion shot (BC, CD, DE) the ball spins fast enough to make it a good gyroscope; but the later the path, the less the spin, and the less will be the effects due to gyroscopic action.

Changes in frictional forces bring in other complications. Static friction is, in general, greater than sliding friction. In Phase Z, there is no sliding. Once the ball enters Phase Z, static friction of higher value has taken over; it tends to gear the ball to the table, and discourage any new tendency to slide.

Sliding occurs in Phases X and Y. As a general proposition found by tests and experience, the higher the sliding velocity, the lower is the frictional force. This is beautifully shown in the first three paths of the nine-cushion shot. Those are high-speed paths (BC, CD, DE) and they show very little loss of velocity. Being high¬speed paths, they are all in Phase X. It may also bo that these paths present an optimum combination of velocity, spin, and axis tilt such that air is trapped under the ball, and that the ball glides partly on air and partly on cloth fibre tips. At any rate, the ball is able to approach its third cushion (E) with plenty of velocity left.

The path EF is slightly curved, and appears to be about uniformly curved. Also, its velocity curve is quite flat. Apparently, EF is an exceptional case in which a whole path from cushion to cushion is made up of Phase Y.

The exceptional path EF can be used to bring forth another comment on Phases X, Y and Z. Remember that the writer's observations of axis positions had to be made within the range of moderate velocities. At high velocity, such as 8 feet per second, the ball moves too fast for close observation. We therefore do not know much about axis tilt, lean and shifting at the high velocities. Remember also that in various shots, different angles of approach to the cushion occur, along with varying combinations of forward velocity, spin, and axis tilt and lean. When these variables are compounded with variations in friction and variations in gyroscopic behaviour due to variations of spin, it becomes evident that a ball may leave a cushion and almost at once take up Phase X, or Phase Y, or possibly even Phase Z.

Different Rates of Loss of Velocity in Phase X.

In contrast with the remarkable conservation of energy displayed in the nine-cushion shot, a severe loss of velocity shows up in the break shot, Fig. 6, 7, 8, in the earlier Phase X periods of both cue ball and object ball. This happens in spite of the high speeds and the (presumably) low frictional effects. What then? Close study of the photographs seem to yield the answer. In the nine-cushion shot, the ball is being cued with considerable english (apparently h = 0.5) and it may be given some follow effect (above center cueing). The presumption then is that between the cue ball's impact with the object ball and the first cushion, and between first and second cushion, ball spin velocity and forward velocity (R and T) are well matched. Whereas, in the break shot, the cue ball was cued almost at center. In its path to the object ball and from there to cushion, the ball is slipping considerably, losing velocity rapidly, acquiring spin, and losing energy in table losses. The object ball starts from impact with almost no spin; and its story for its first two paths is much the same as for the object ball.

The Ball in Natural Flight.

The properties and tendencies of the ball in natural flight are of the greatest importance in three-cushion billiards. Many times, the. "leave" from the last shot presents a situation which may be shot in two or three different ways. (As an extreme, the writer is able to take care of one certain leave by shooting it in about 14 different ways). When he can do so, the player strongly tends to play that shot which is a natural - in which the cue ball will follow a natural or nearly natural path. Reasons: the ball carries well, and its natural flights are more easily learned and standardized than are most unnatural flights.

The properties of natural flight enable a player to go long distances and still make his shot. The casual observer tends to give the player great credit for some of the spectacular long-distance shots. The player smiles to himself, for he knows they are often far easier than some short little unnatural shots that may lock simpler and easier. All this is made clearer by considering the nine-cushion shot, in which the cue ball travels over 40 feet; or a very ordinary seven-cushion shot, when it may easily go 30 feet. The several tendencies of the ball in natural flight, at cushion and on table, tend to iron out slight initial errors made in judgment or execution when cueing the ball. In contrast, think of an imaginary straight-ahead 40-foot shot on a table 40 feet long. Put the object ball at one end, and the player and cue ball at the other. All the player has to do is to shoot straight ahead for 40 feet and hit the ball at the far end. Well, nothing will happen in this cue ball's flight to correct any error. It has to be shot right, or it will be a clean miss. It is doubtful if Hoppe himself could make the shot more than half the time.

Natural flight is an optimum condition, in which greatest reliability and simplicity of a long course is combined with least energy losses. It is a very remarkable combination.

The Diamond System.

The Diamond System is based on the simple properties of natural flight. The markers, or "diamonds", are placed 14 inches apart (very nearly) on the standard full-size table. They are put there to be used. By means of a simple numerical system, the expert can predict the course of the ball with very good accuracy; and the amateur can do far better with it than without it. In any case, one must standardize the speed of stroke and the amount of running english used. The system is completely explained in Hoppe's book.

When the phrase was used above,- "the simple properties of natural flight",- the reader may have remembered the complex arguments about Phases X, Y and Z and found himself puzzled. The point is, that the ball can and does do some rather complicated things within a single path of a natural course. It is all the more interesting, therefore, to know that because of these very complications, the lines and angles and cushion contact points of the paths themselves boil down to a simple numerical system of over-all behaviour.

Cushion Height.

The height of the cushion edge is, of course, vitally related to natural flight phenomena. The cushion touches the standard ball at a point 1 7/16 inches above table surface, or about one-fourth inch above center. The writer has not been able to unearth any reason as to why this height was adopted. The known history of the game does tell us that the cue evolved by a painful trial-and-error process. No doubt, cushion edge height did also. One suspects that the present height has been worked out by experience, to be the best height for natural flight. At any rate, this is true: a player who is used to using standard balls will miss many shots when he has to use over- or under-size balls. The natural course of the ball is changed.

SECTION V. FURTHER NOTES.

The Bridge.

The word bridge has two meanings in billiards. It means the mechanical support and guidance given to the cue by the left hand's fingers and thumb; it also means the distance from the bridge hand to the cue ball,- or the pre-impact accelerating part of the stroke. The experts lay great stress on both factors.

Some shots have to be played with an open bridge (as in shooting over a ball) in which case the cue slides over an open V between thumb and forefinger. But Mr. Hoppe has told the writer that he knows of no expert who ever uses an open bridge when it is possible to use a closed bridge. In the closed bridge, the forefinger is curled around the cue; and it, together with the thumb and the flesh of the hand and the cue contact with the middle finger, makes a snugly fitting flesh-lined hole through which the cue must slide. There are several variations of arranging the bridge hand, depending on circumstances.

The bridge affects the mechanics of the game in several ways. It has three obvious functions. First, it enables the player to fix upon the point at which he will cue the ball; this is when he "addresses" the ball with some trial strokes. Second, the bridge is a guide which helps to make sure that the stroke will be going straight ahead at the time of impact. Third, changing the bridge distance changes the space for the accelerating part of the stroke - thereby permitting the player to change the cue velocity at impact without much variation in accelerating force.

There may be a fourth function, concerned with vibration due to the initial shock of impact. In mechanics, it is known that elastic bodies, during impact, exhibit vibration due to the initial shock. The initial shock probably tends to knock the cue tip away from the ball, when the cueing is off-center, and to cause a miscue. The flesh surrounding the shaft of the cue should help to dampen out crosswise vibrations of the front end of the cue due to the initial shock. As impact proceeds, the cue then has a chance to "get its teeth" into the ball, to drive the ball straight ahead. The writer suspects that this fourth function of the bridge may become recognized as the main reason for using the closed rather than the open bridge.

As the cue slides through, there is frictional force at the bridge; furthermore, at the start, the bridge encloses a shaft diameter of one-half inch — and at the finish of a long stroke, the enclosed shaft diameter is about one inch. What will all this do to the mechanics analysis of Hoppe's stroke, made in Section III? In the writer's opinion, it does very little. The bridge is not tight; it is merely snug. Little frictional force is present. And as to the cue getting larger as it slides through - the player soon learns all about that, and automatically relaxes the snugness of the bridge as the stroke progresses.

One-Handed Billiards.

When the cue ball is within a foot or less of the cushion, it is possible to make many three-cushion shots without using the bridge hand. The cue is simply rested on the rail. While the studies cited in this paper were in progress, the writer began to experiment with one-handed shots,- partly out of curiosity, and partly to learn more about cue-to-ball impact. The results were most surprising. Proper handling of the cue was very quickly learned. Making bank shots (sending the cue ball alone to touch three cushions before it touches the two other balls, which are usually close together for such a shot) turned out to be more successful, under certain circumstances, with one hand in use than with two.

Two strong arguments support the one handed bank shot. First, one stands erect, thereby being better able to judge the angles and the expected course of the ball. Second, one is forced to use a standard stroke: too easy a stroke, and the ball dies on the course: too hard a stroke, and a miscue results. Plenty of other ordinary shots can be reliably performed one-handed. This variation of the game is most fascinating.

To secure reliable results, the cue must be gripped an inch or two back of the balance, or center of gravity of the cue. The reason for this will appear below.

Grip Location.

The place where the right hand grips the cue affects the mechanics of the game. A cue is made with a large diameter butt. Approximately the last foot of the butt is sometimes covered with leather, or a thread wrapping. The amateur naturally thinks that this covering is put there to be used by the grip. It is well to remember that the man who makes a cue may not himself be a player; and even if he is, he is probably an amateur who grips the cue in the wrong place. On most covered-butt cues the writer has seen, the covering does not reach forward to the place where the grip should nearly always be located. Hence, the writer is very much puzzled about location of grip coverings.

As Hoppe makes clear in his book, the grip should nearly always be an inch or two behind the balance, or even at the balance for some shots. The amateur nearly always is too far back. The main reason for using proper grip location has to do with impact. When the ball is cued off-center, a component of impact force tends to throw the shaft out sidewise from the ball. The closed bridge, in part, helps to dampen out this effect. But also, if it is desired to have the action occur with the least shivering of the cue shaft, the cue hand should be supporting the cue at the center of spontaneous rotation; and that will be at a place not far behind the balance point.

As an instance, a certain player who plays a very good game, saw the writer making three-cushion follow shots: the cue ball hits the object nearly full, then follows on goes all the way around the table because it was cued above center, and makes the shot. The player wanted to know how to do it. The writer observed the player as he took his usual stance and grip. The cue hand was several inches too far back. The writer (who only that day had read Hoppe's advice on this point and himself was just in the act of mastering it for the first time) merely had the player shift his cue hand forward to near the balance point. The player at once began making these shots - for the first time in his 25 years of play.

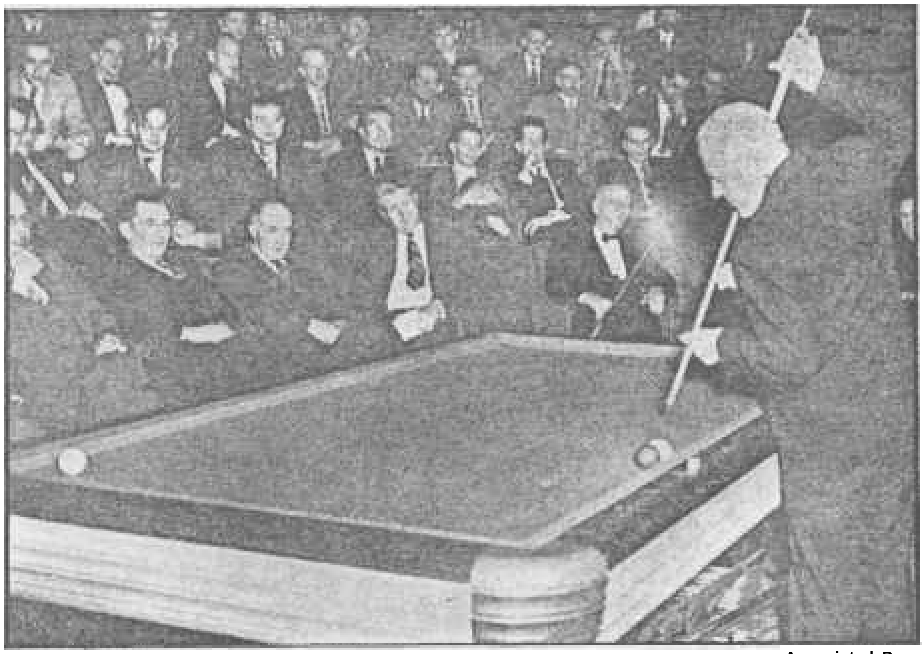
Magnus Effect.

The Magnus effect must be mentioned. It is the effect of the air upon a spinning ball in flight, that either does or does not cause a baseball to curve. The baseball controversy will not be argued here, but we can settle the billiard argument before it even gets started. The theory has been worked out for cylinders, and cylinders have also been tested experimentally. The case of the sphere apparently has not been worked out or tested. Using data from the cylinder, and making assumptions to fit the sphere, the writer computed the Magnus effect for these conditions: ball moving ahead at 3 feet per second, with a spin velocity at its equator of the same amount. The figures show a possible value of around 0.0005 lbs. crosswise force. The writer's conclusion is this: Magnus affect is certainly present, but no conditions ever occur in billiards in which the effect is large enough to be worth taking into account.

Some readers in the field of aerodynamics may be inclined to suggest that "ground effect" may add considerably to the force one otherwise gets, That is, the ball moves over a plane, and is not a free body in air. To test that possibility, the writer hung a rubber ball up by a long thread. Pre-twisting the thread gave the ball a good spin as it moved back and forth as a pendulum. A gently curved piece of Masonite was moved against the path of the ball. If Magnus plus ground effect were present in considerable degree, the ball would have tried to hug the curve. It did not. Whereas, with the Masonite removed, simple Magnus effect did somewhat curve the path of the light rubber ball.

Finally.-

This question will occur to many: will a better understanding of the mechanics of the game improve a player's performance? In the writer's opinion, the answer is unequivocally "Yes". However, this paper was not written with the hope of raising the general standards of billiard play. But there actually is a great deal of curiosity abroad as to why billiard balls behave the way they do. It is hoped that the paper will satisfy that curiosity in part, for those who happen to read it.



Associated Press

WILLIE HOPPE AT THE UNIVERSITY OF MICHIGAN
Nine cushions and the coefficient of restitution.

Poolroom Science

In an amphitheater at the University of Michigan, the eyes of 200 scientists were focused on a billiard table. The greatest billiard player of them all, Willie Hoppe, dressed in a dinner jacket and cool as a master surgeon about to operate, stood ready. But first there was a lecture from Engineering Professor Arthur Moore, a billiard player for 30 years, on his six years' experiments to make a science out of a sport. Willie Hoppe's English on the ball was not less understandable than Professor Moore's English on the theory.

The professor, author of a 41-page thesis on the subject, defined bouncibility as the "coefficient of restitution," and divided all players into two groups: amateurs, who use a "ballistic" or shoving stroke, and professionals, who use a smooth, controlled stroke, with a follow-through.

High spot of the evening was Willie Hoppe's famed nine-cushion shot, in which the ball travels more than 40 feet. What baffled Professor Moore was that on the sixth and eighth cushions, the ball -both lost and gained velocity. The fact is, Professor Moore discovered, that when Hoppe cued the ball with English—as any poolroom fan could have told him, though not in so many words—he gave the ball rotational energy as well as its usual translational or rolling energy. When the ball's spin slowed, the energy was turned into forward roll.

With facts & figures, Professor Moore demonstrated that the technique most good players use is scientifically superior: the pendulum stroke, with forearm swinging vertically from the elbow. Unfortunately for Professor Moore's thesis, Willie uses a side-arm stroke. It was a habit he picked up lying belly-to-billiard-table as a boy of five. Said 59-year-old Willie Hoppe: "It's too complicated for me. I guess this analysis came too late to help my game."