TP B.24
Estimating Rolling CB Carom Angle with 1/4 Cue Point

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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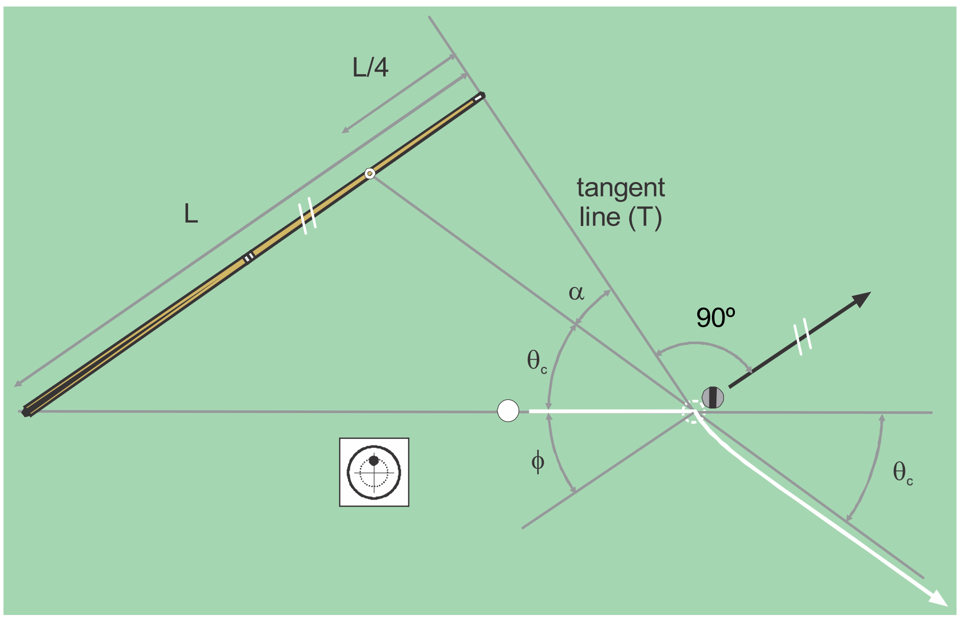


The CB carom angle (θ_c) can be approximated for any cut angle (ϕ) by pointing the cue in the OB direction with the tip at the ghost ball and shifting the cue parallel up the tangent line (distance T) until the butt end is along the aiming line. A line from the L/4 point on the cue through the ghost ball will approximate the carom angle. This is based on the theoretical approach diagrammed on page 7 in TP A.4. The change in ratio from 2/7 to 1/4 is a simplification since half of half (1/4) is easy to visualize (although, any point on the cue can be marked). Below are the equations describing the variables in the diagram above, where the carom line originates from a factor (f) of the length (1/4 in the diagram). An option to using the entire cue is just use the shaft, with the joint on the aiming line instead of the butt, with the carom line again going through the length-factor point (e.g., 1/4 of the shaft length).

$$\tan(\phi) = \frac{T}{L} \qquad \tan(\alpha) = \frac{f \cdot L}{T} \qquad \theta_c = 90 \cdot \deg - \phi - \alpha$$

So the CB carom angle can be approximated by:

$$\theta_{ca}(\phi, f) := 90 \cdot \deg - \phi - \operatorname{atan}\left(\frac{f}{\tan(\phi)}\right) \qquad f := \frac{1}{4}$$

From TP A.6, the theoretical CB carom angle, accounting for ball inelasticiy and friction, is:

$$e := 0.94 \qquad \mu_{balls} := 0.06 \qquad v := 2 \qquad R := \frac{\frac{2.25}{2}\cdot in}{m}$$

$$\omega_{x0}(v,\phi,e,\mu_{balls}) := \frac{v}{R}\cdot\left[\frac{5}{4}\cdot\mu_{balls}\cdot(1+e)\cdot\cos(\phi)^3 - 1\right]$$

$$\omega_{y0}(v,\phi,e,\mu_{balls}) := \frac{v}{R}\cdot\left[\left[\frac{5}{4}\cdot\mu_{balls}\cdot(1+e)\cdot\sin(\phi)\cdot\cos(\phi)^2\right]\right]$$

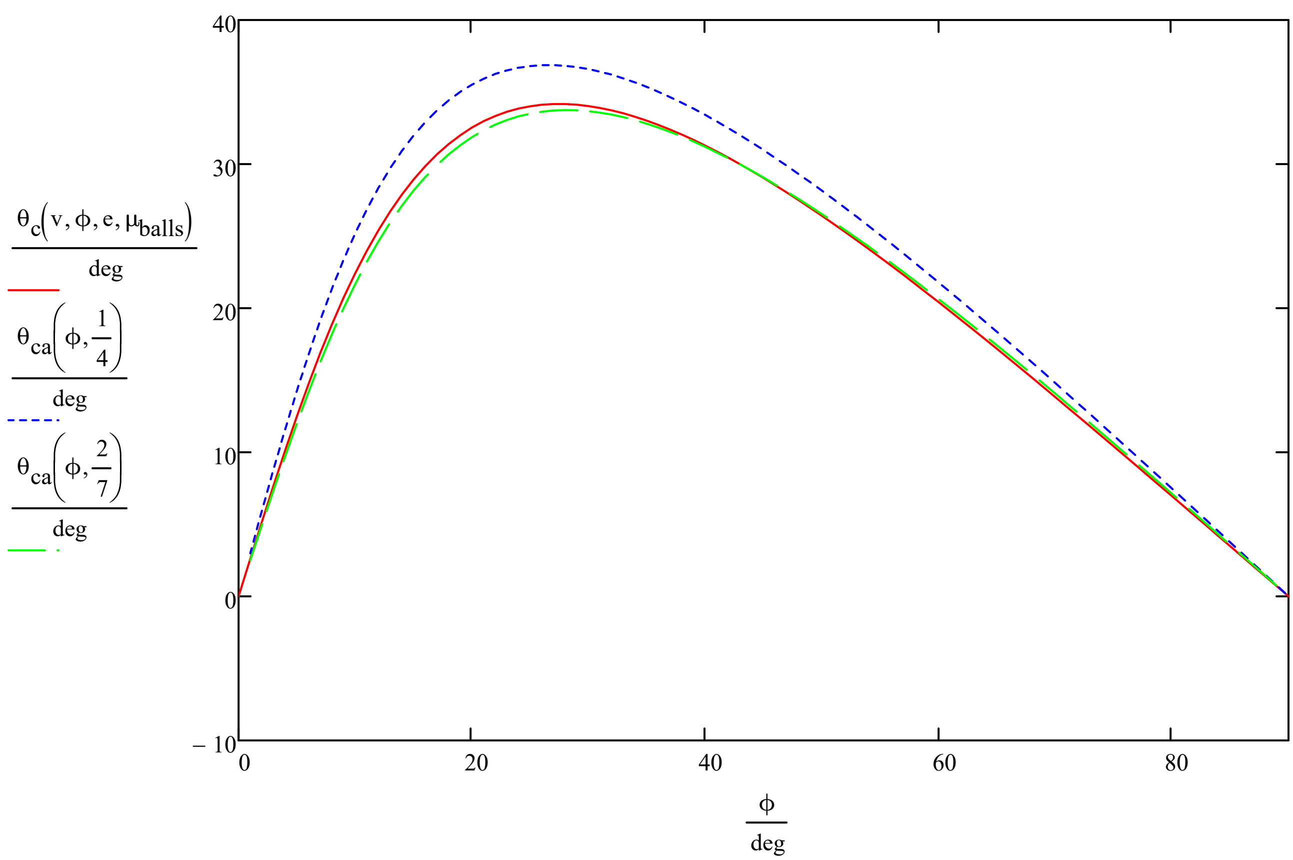
$$v_{x0}(v,\phi,e,\mu_{balls}) := \frac{v}{2}\cdot\sin(\phi)\cdot\cos(\phi)\cdot\left[1 + e - \mu_{balls}\cdot(1+e)\cdot\cos(\phi)\right]$$

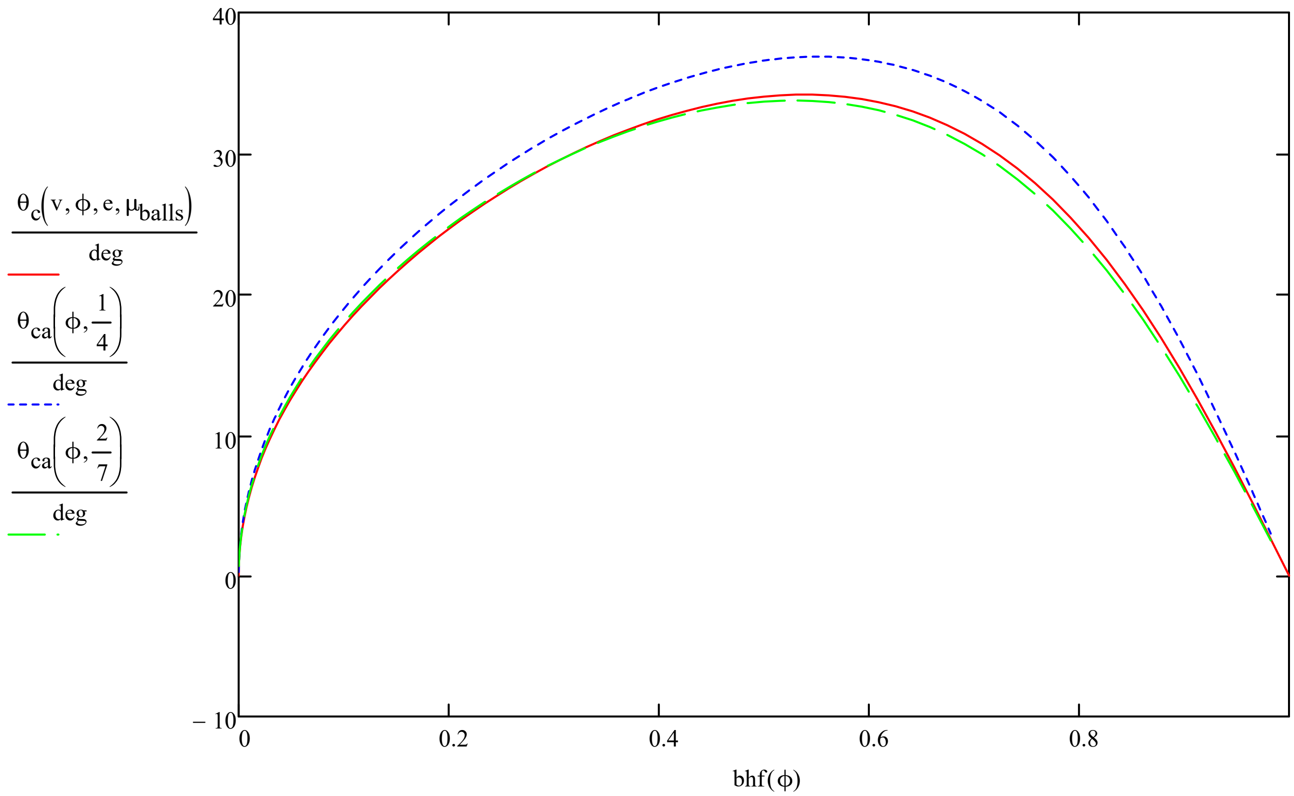
$$v_{y0}(v,\phi,e,\mu_{balls}) := \frac{v}{2}\cdot\left[\sin(\phi)^2\cdot\left[2 - \mu_{balls}\cdot(1+e)\cdot\cos(\phi)\right] + (1-e)\cdot\cos(\phi)^2\right]$$

$$\theta_c(v,\phi,e,\mu_{balls}) := \operatorname{atan}\left(\frac{5\cdot v_{x0}(v,\phi,e,\mu_{balls}) + 2\cdot R\cdot\omega_{y0}(v,\phi,e,\mu_{balls})}{5\cdot v_{y0}(v,\phi,e,\mu_{balls}) - 2\cdot R\cdot\omega_{x0}(v,\phi,e,\mu_{balls})}\right)$$

Here's how cue ball carom angle varies with cut angle and ball-hit fraction (bhf) for length factors of 1/4 (approximation) and 2/7 (ideal with no inelasticity or friction):

$$\phi := 0\cdot deg, 1\cdot deg .. 90\cdot deg \qquad bhf(\phi) := 1 - \sin(\phi)$$

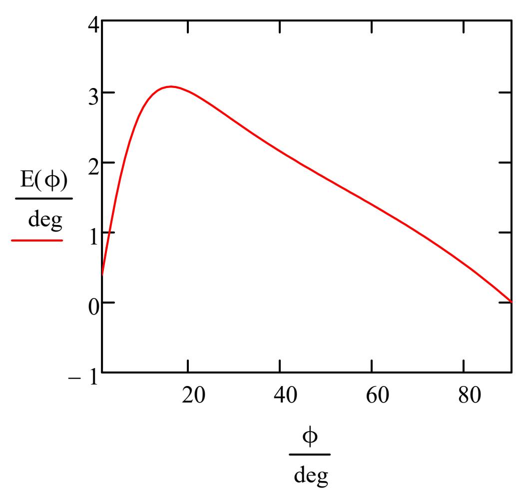




The ideal 2/7 length factor obviously provides a better match than the 1/4 approximation, but the approximation works fairly well over all cut angles.

Here is the error in the 1/4 approximation as compared to the actual CB carom angle:

$$E(\phi) := \theta_{ca}\left(\phi, \frac{1}{4}\right) - \theta_c\left(v, \phi, e, \mu_{balls}\right)$$



$\phi := 20 \cdot deg$ initial guess

$\phi_{max} := \text{maximize}(E, \phi) = 15.886\,deg$ $\quad bhf\left(\phi_{max}\right) = 0.726$ $\quad E\left(\phi_{max}\right) = 3.06\,deg$

If you want to mark your cue or shaft with the ideal length factor to match the carom angle well over a wide range, use:

$$\phi := 30 \cdot deg \qquad f := \frac{1}{4} \qquad E(f) := \theta_{ca}(\phi, f) - \theta_c\left(v, \phi, e, \mu_{balls}\right)$$

$$f_{opt} := root(E(f), f) \qquad f_{opt} = 0.281$$

So if using the entire cue of standard length, the carom line point should be at:

$$L := 58 \cdot in \qquad f_{opt} \cdot L = 16.3\,in$$

$$L = 147.3\,cm \qquad f_{opt} \cdot L = 41.4\,cm$$

If only using only a standard-length shaft or butt, the carom line point should be at:

$$L := 29 \cdot in \qquad f_{opt} \cdot L = 8.2\,in$$

$$L = 73.7\,cm \qquad f_{opt} \cdot L = 20.7\,cm$$

As you can see below, the approximated carom angle matches the actual very well over all cut angles:

$$\phi := 0 \cdot deg, 1 \cdot deg \,..\, 90 \cdot deg$$