technical proof

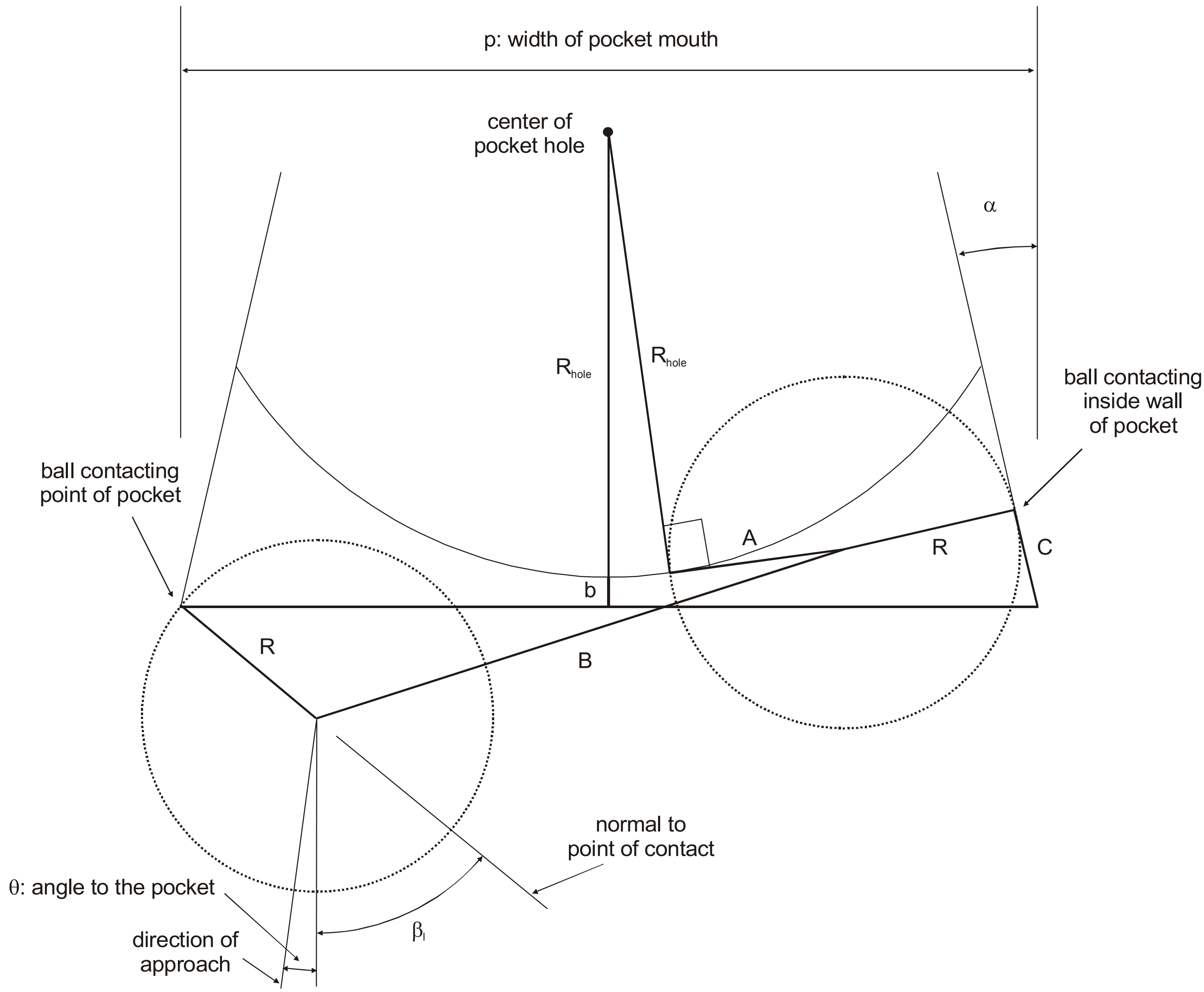
technical proof

TP 3.5
Effective target sizes for slow shots into a side pocket at different angles

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 7/4/2003 last revision: 1/22/2023



Ball radius:

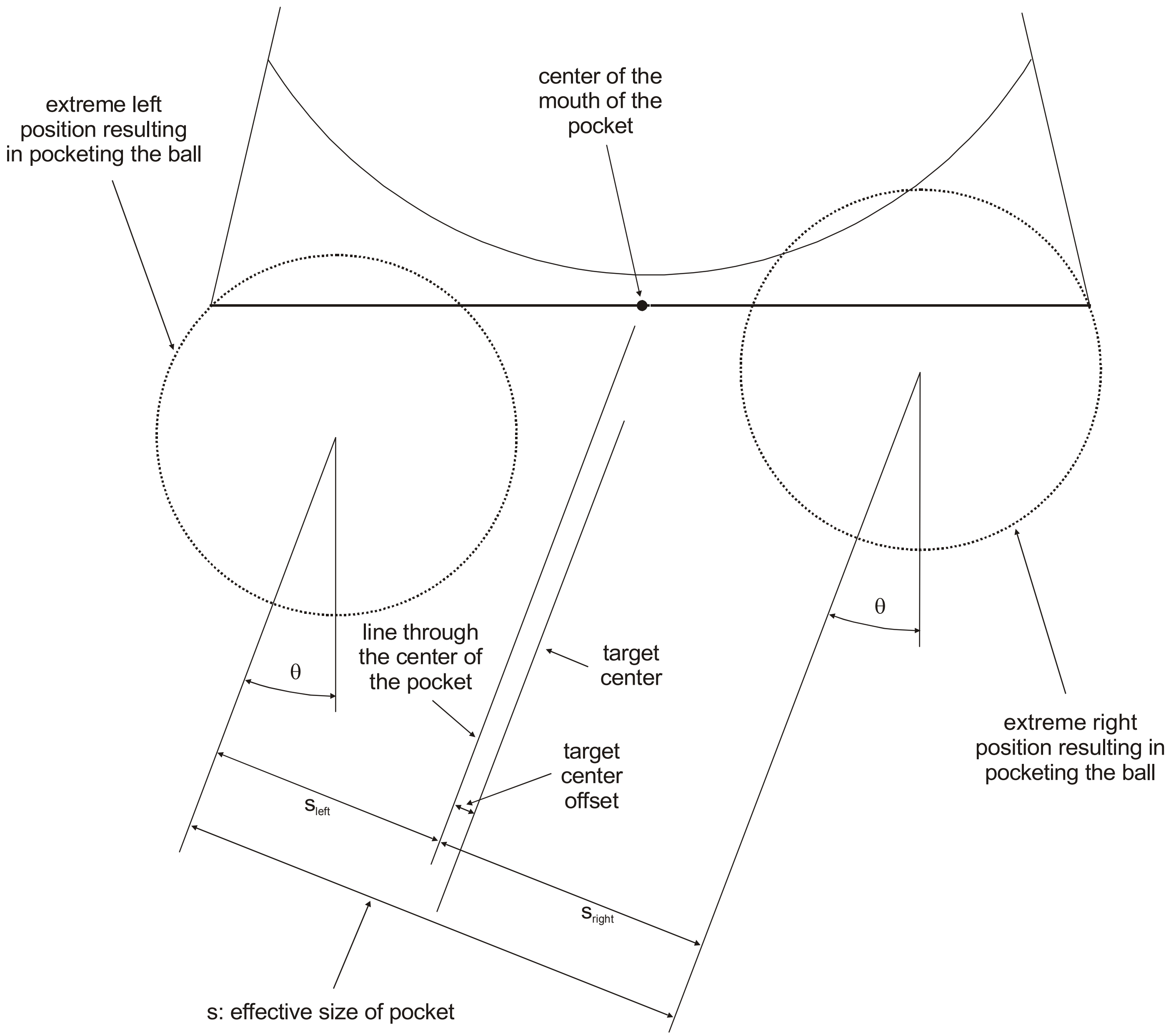
$R := 1.125$

Pocket-specific parameters:

p: mouth width α: wall angle R_{hole}: hole radius b:shelf depth to hole

$p := 5.0625$ $\alpha := 14 \cdot deg$ $R_{hole} := 3$ $b := 0.1875$

The equations below are derived from the figure above by writing vector loop equations around the path shown bold. The pocket target size is defined by the figure below.



General equations for the size of left portion of target area assuming point deflection

$$A(\alpha, p, b, R_{hole}, \beta_l, \theta) := \frac{1}{\sin(2\cdot\beta_l - \theta - \alpha)}\cdot\left[\frac{p}{2}\cdot\cos(\alpha) - R - R_{hole}\cdot\cos(2\cdot\beta_l - \theta - \alpha) - (R_{hole} + b)\cdot\sin(\alpha)\right]$$

$$poly_{\beta}(\alpha, p, b, R_{hole}, \beta_l, \theta) := R\cdot\sin(\beta_l) + A(\alpha, p, b, R_{hole}, \beta_l, \theta)\cdot\sin(4\cdot\beta_l - 2\cdot\theta - 2\cdot\alpha) + R_{hole}\cdot\cos(4\cdot\beta_l - 2\cdot\theta - 2\cdot\alpha) \ldots + \frac{p}{2}\cdot\cos(2\cdot\beta_l - \theta) + (R_{hole} + b)\cdot\sin(2\cdot\beta_l - \theta)$$

$$poly_{\beta\_in}(p, b, R_{hole}, \beta_l, \theta) := -R\cdot\sin(\beta_l) + R_{hole} - \frac{p}{2}\cdot\cos(2\cdot\beta_l - \theta) - (R_{hole} + b)\cdot\sin(2\cdot\beta_l - \theta)$$

$$\beta_{guess}(\theta) := \left[20\cdot deg + \frac{70}{130}\cdot(\theta + 60\cdot deg)\right] \qquad \beta := \beta_{guess}(0\cdot deg) \qquad \beta = 52.308\,deg$$

$$\beta_l(\alpha, p, b, R_{hole}, \theta) := \min\left(\begin{pmatrix} root(poly_{\beta}(\alpha, p, b, R_{hole}, \beta, \theta), \beta) \\ root(poly_{\beta\_in}(p, b, R_{hole}, \beta, \theta), \beta) \end{pmatrix}\right)$$

$$s_{left\_point}(\alpha, p, b, R_{hole}, \theta) := \frac{p}{2}\cdot\cos(\theta) - R\cdot\sin(\beta_l(\alpha, p, b, R_{hole}, \theta))$$

Size of the left portion of the target area assuming one wall deflection

$$r_{wall\_1}(\alpha, p, b, R_{hole}, \theta) := A(\alpha, p, b, R_{hole}, 90\cdot deg, \theta)\cdot\sin(2\cdot\theta + 2\cdot\alpha) - R_{hole}\cdot\cos(2\cdot\theta + 2\cdot\alpha)$$

$$r_{wall\_2}(p, b, R_{hole}, \theta) := \frac{p}{2}\cdot\cos(\theta) - (R_{hole} + b)\cdot\sin(\theta)$$

$$r_{wall}(\alpha, p, b, R_{hole}, \theta) := r_{wall\_1}(\alpha, p, b, R_{hole}, \theta) + r_{wall\_2}(p, b, R_{hole}, \theta)$$

$$s_{left\_wall}(\alpha, p, b, R_{hole}, \theta) := -\left(\frac{p}{2}\cdot\cos(-\theta) - r_{wall}(\alpha, p, b, R_{hole}, -\theta)\right)$$

Maximum angle for slight near point deflection and rattle-in

$\theta := 70\cdot deg$

Given

$poly_{\beta}(\alpha, p, b, R_{hole}, 90\cdot deg, \theta) = 0\cdot deg$

$\theta_{max} := Find(\theta) \qquad \theta_{max} = 68.292\,deg \qquad \theta_{max} := root(poly_{\beta}(\alpha, p, b, R_{hole}, 90\cdot deg, \theta), \theta) \qquad \theta_{max} = 68.292\,deg$

equation check: $r_{wall}(\alpha, p, b, R_{hole}, \theta_{max}) = 1.125 \qquad R = 1.125$

Critical angle between far point rattle-in and far wall deflection

$\theta := -51.4689\cdot deg \qquad \beta := 25\cdot deg$

Given

$poly_{\beta\_in}(p, b, R_{hole}, \beta, \theta) = 0$

$\beta := Find(\beta) \qquad \beta = 26.036\,deg \qquad \beta := \beta_l(\alpha, p, b, R_{hole}, \theta) \qquad \beta = 26.036\,deg$

$\theta - \alpha + 90\cdot deg - \beta = -1.505\,deg$

$\theta_{critical} := \beta + \alpha - 90\cdot deg \qquad \theta_{critical} = -49.964\,deg$

Minimum angle for far wall deflection

$\theta_{min} := -\theta_{max} \qquad \theta_{min} = -68.292\,deg$

Size of the left portion of the target area

$$s_{left}(\theta) := \begin{cases} 0 & \text{if } \theta \geq \theta_{max} \\ 0 & \text{if } \theta \leq \theta_{min} \\ s_{left\_wall}(\alpha, p, b, R_{hole}, \theta) & \text{if } \theta_{min} < \theta < \theta_{critical} \\ s_{left\_point}(\alpha, p, b, R_{hole}, \theta) & \text{otherwise} \end{cases}$$

$$s_{right}(\theta) := s_{left}(-\theta)$$

$$s(\theta) := s_{left}(\theta) + s_{right}(\theta)$$

$$offset(\theta) := \frac{(s_{right}(\theta) - s_{left}(\theta))}{2}$$

Plot results:

$$N := 50 \qquad i := 0..N \qquad \theta_i := (-\theta_{max} + 1\cdot deg) + \frac{2\cdot(\theta_{max} - 1\cdot deg)}{N}\cdot i$$

$$\beta_i := \beta_{guess}(\theta_i) \qquad \beta_{in} := \beta$$

Given

$$poly_{\beta}(\alpha, p, b, R_{hole}, \beta, \theta) = 0$$

$$\beta := Find(\beta)$$

Given

$$poly_{\beta\_in}(p, b, R_{hole}, \beta_{in}, \theta) = 0$$

$$\beta_{in} := Find(\beta_{in})$$

$$\beta_i := \min\left(\begin{pmatrix} \beta_i \\ \beta_{in_i} \end{pmatrix}\right)$$

$$s_{left\_point_i} := \frac{p}{2}\cdot\cos(\theta_i) - R\cdot\sin(\beta_i)$$

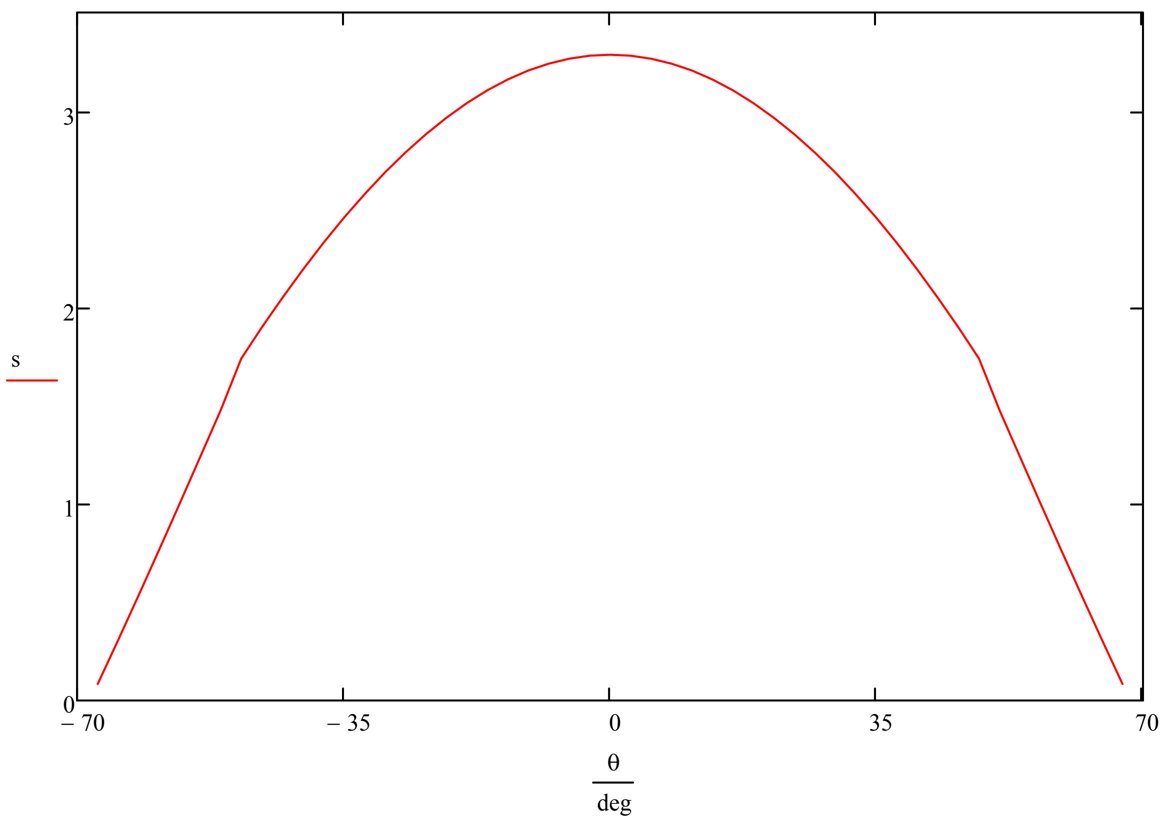
$$r_i := r_{wall}(\alpha, p, b, R_{hole}, -\theta_i)$$

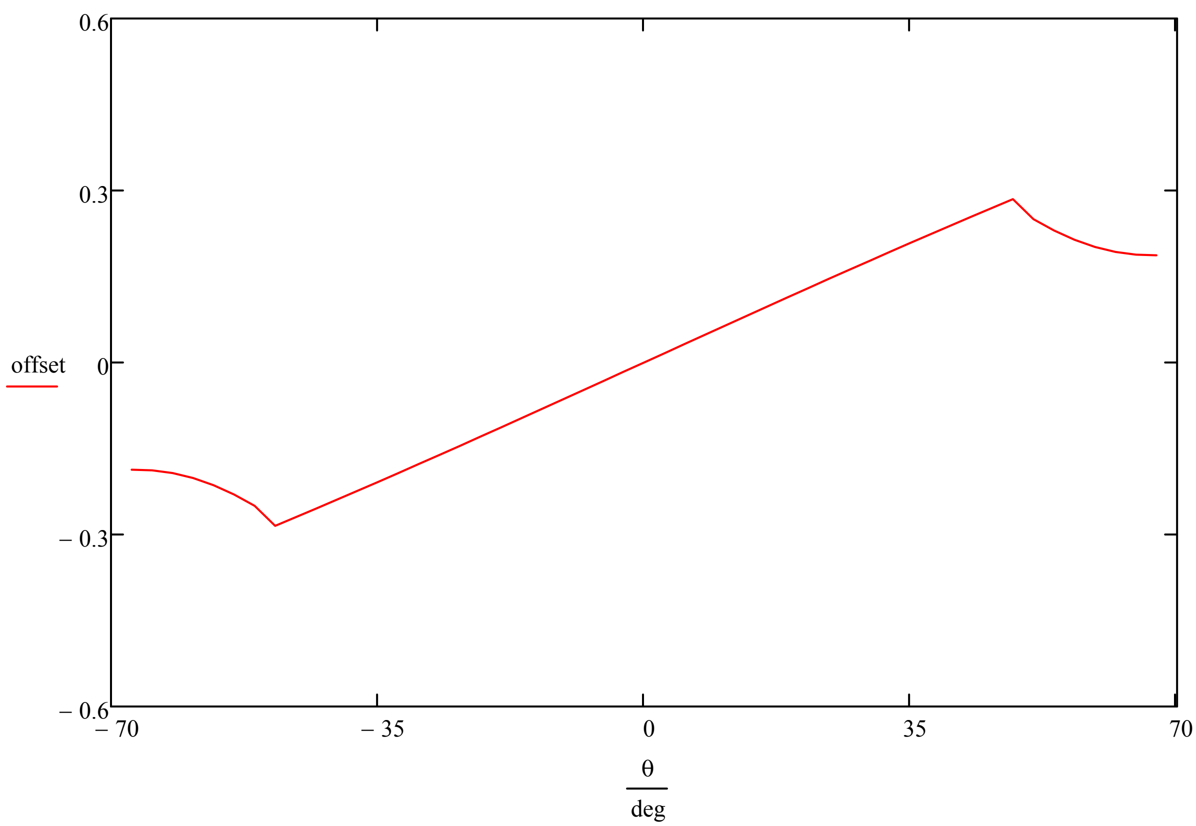
$$s_{left\_wall_i} := -\left(\frac{p}{2}\cdot\cos(-\theta_i) - r_i\right)$$

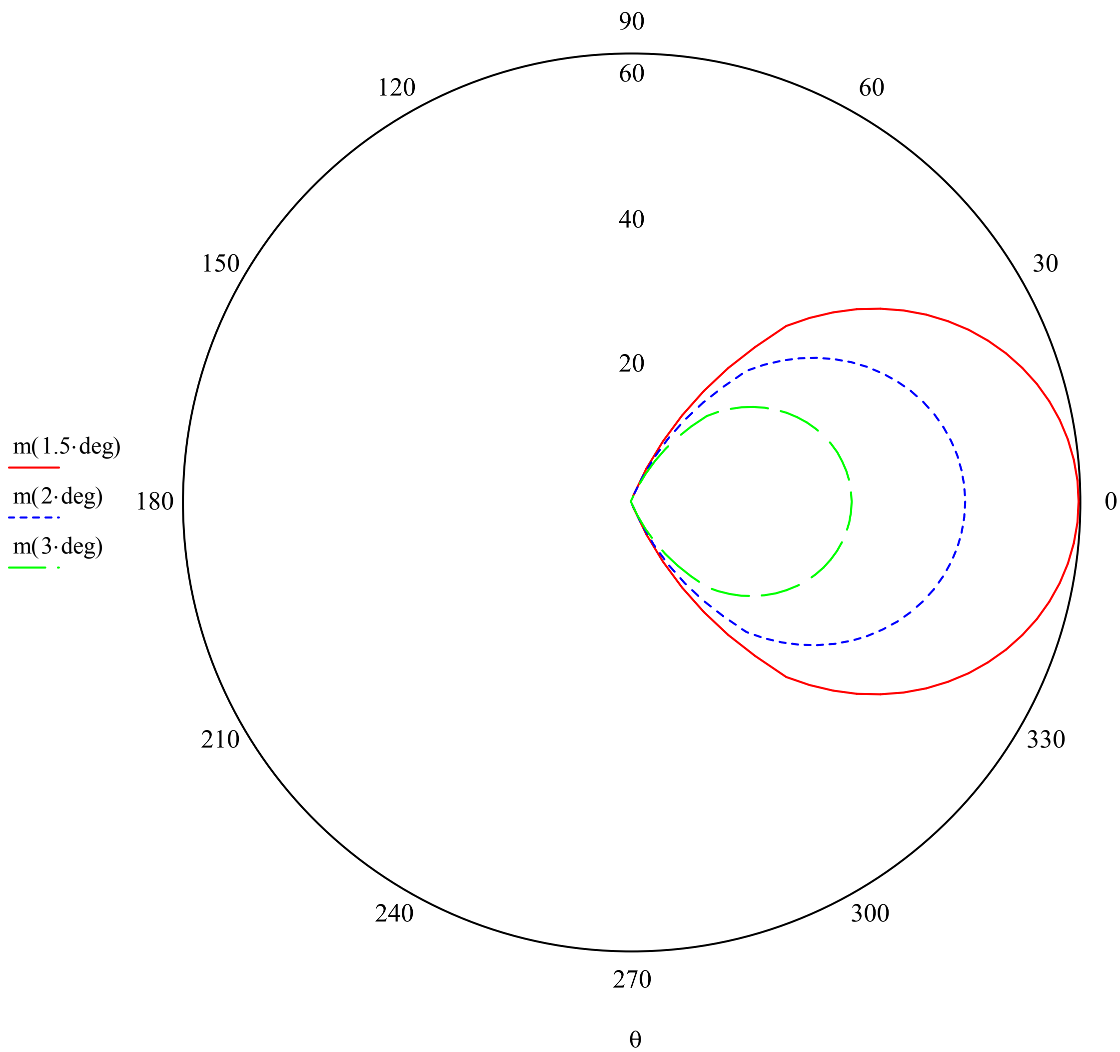
$$s_{left_i} := if(\theta_i > \theta_{max}, 0, if(\theta_i < \theta_{min}, 0, if(\theta_i < \theta_{critical}, s_{left\_wall_i}, s_{left\_point_i})))$$

$$s_{right_i} := s_{left_{N-i}} \qquad s := s_{left} + s_{right} \qquad offset := \frac{(s_{right} - s_{left})}{2}$$

$$m(\Delta\theta) := \frac{1}{2\cdot\tan(\Delta\theta)}\cdot s$$


3
2
1
0
s
− 70
− 35
0
35
70
θ
deg


0.6
0.3
0
− 0.3
− 0.6
offset
− 70
− 35
0
35
70
θ
deg


90
60
120
60
40
150
30
20
m(1.5·deg)
m(2·deg)
180
0
m(3·deg)
210
330
240
300
270
θ