technical proof technical proof

TP 3.3
30° rule

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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originally posted: 7/6/2003 last revision: 10/2/2010

An complete derivation, arrived at by solving the general equations of motion, can be found in TP A.4.

Ball-hit fraction:

$$f := 0, 0.01 .. 1$$

Cut angle:

$$\varphi(f) := \operatorname{asin}(1 - f)$$

Cue ball final deflected angle:

$$\theta(\varphi) := \operatorname{atan}\left(\frac{\sin(\varphi)\cdot\cos(\varphi)}{\sin(\varphi)^2 + \frac{2}{5}}\right)$$

For a half-ball hit:

$$\varphi\left(\frac{1}{2}\right) = 30\,\text{deg}$$

$$\theta\left(\varphi\left(\frac{1}{2}\right)\right) = 33.67\,\text{deg}$$

For a 1/4-ball hit:

$$\varphi\left(\frac{1}{4}\right) = 48.59\,\text{deg}$$

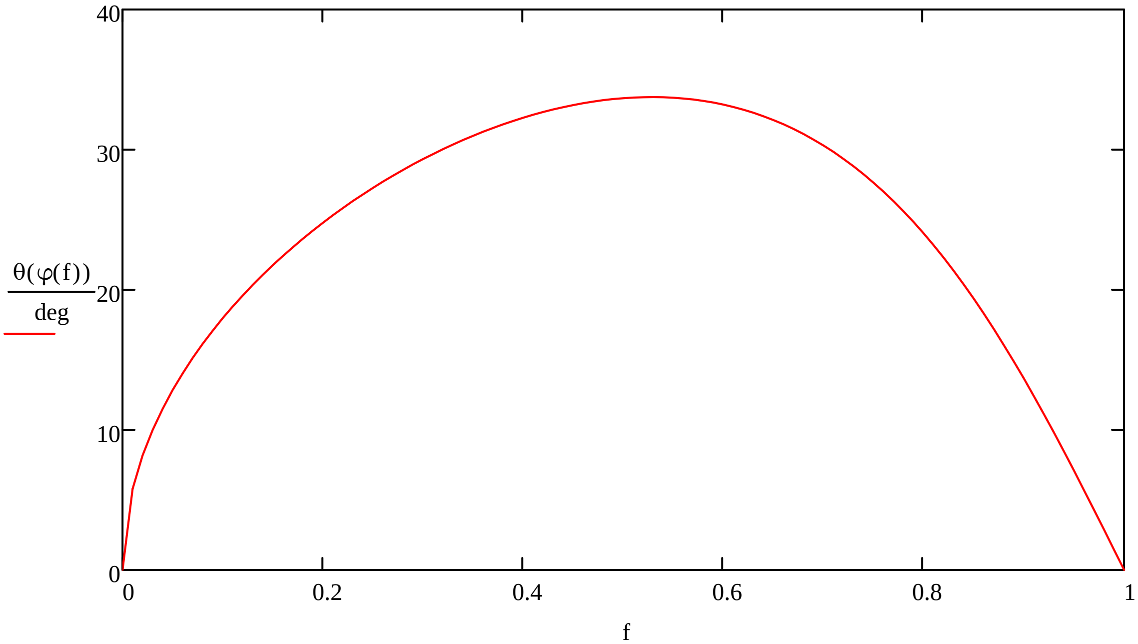
$$\theta\left(\varphi\left(\frac{1}{4}\right)\right) = 27.267\,\text{deg}$$

For a 3/4-ball hit:

$$\varphi\left(\frac{3}{4}\right) = 14.478\,\text{deg}$$

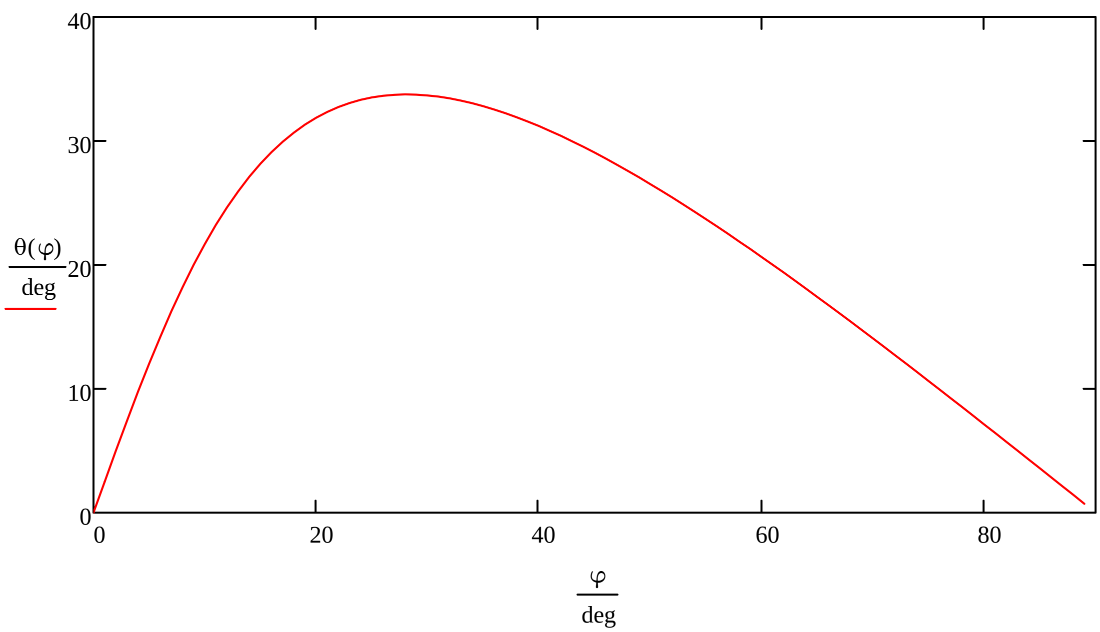
$$\theta\left(\varphi\left(\frac{3}{4}\right)\right) = 27.626\,\text{deg}$$

deflected cue ball angle vs. ball-hit fraction:



deflected cue ball angle vs. cut angle:

$\varphi := 0\cdot deg, 1\cdot deg .. 89\cdot deg$



Maximum cue ball deflected angle:

Deflected angle as a function of cut angle is:

$$\operatorname{atan}\left(\frac{\sin(\varphi)\cdot\cos(\varphi)}{\sin(\varphi)^2 + \frac{2}{5}}\right)$$

The derivative of this with respect to φ (using MathCAD) is:

$$-\left(\frac{45\cdot\sin(\varphi)^2 - 10}{45\cdot\sin(\varphi)^2 + 4}\right)$$

At the maximum, the numerator of this expression must be 0, so:

$$\sin(\varphi) = \sqrt{\frac{10}{45}} = \frac{\sqrt{2}}{3}$$

Therefore, the maximum cue ball deflection occurs at a cut angle of:

$$\varphi := \operatorname{asin}\left(\frac{\sqrt{2}}{3}\right) = 28.126\,\text{deg}$$

which corresponds to a ball-hit fraction of:

$$f(\varphi) := 1 - \sin(\varphi) \qquad f(\varphi) = 0.529$$

and the maximum deflected angle is:

$$\theta(\varphi) = 33.749\,\text{deg}$$