technical proof

TP 2.1
Minimum cue tip friction required for no-slip horizontal impact

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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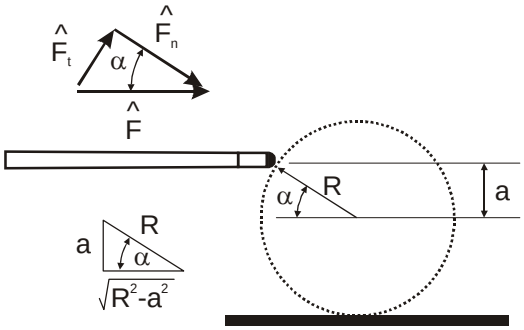
ball size: $D := 2.25 \cdot in$ $R := \frac{D}{2}$



Note: in this simple analysis, the effects of squirt are not considered. The impulse of the cue is assumed to be in the cue direction. In reality, the exact angle (and the effective tip offset) varies with the amount of English and the type of cue. However, squirt angles are typically less than 2 degrees, so the effects are small.

Horizontal impulse between the cue stick and cue ball:

$$F'$$

Normal impulse (perpendicular to the ball):

$$F'_n = F' \cdot \cos(\alpha) \tag{1}$$

Tangential impulse (tangent to the ball):

$$F'_t = F' \cdot \sin(\alpha) \tag{2}$$

On the verge of slip, using Equation 1:

$$F'_t = \mu \cdot F'_n = \mu \cdot (F' \cdot \cos(\alpha)) \qquad (3)$$

Equating Equations 2 and 3 gives:

$$\mu = \frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha) = \frac{a}{\sqrt{R^2 - a^2}} \qquad \mu(a) := \frac{a}{\sqrt{R^2 - a^2}}$$

Typical values for the coefficient of friction required to avoid a miscue:

at the generally recommended maximum tip offset:

$$\mu(0.5 \cdot R) = 0.577$$

at typical measured maximum effective offsets (e.g., see my October '05 article):

$$\mu(0.55 \cdot R) = 0.659$$